Name: Richard

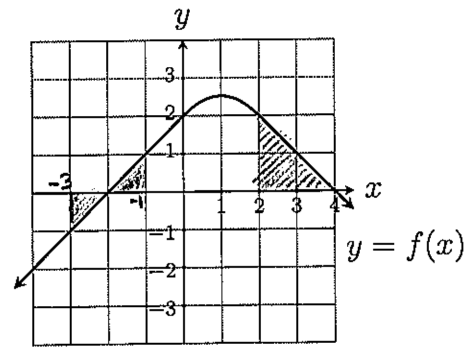
QUIZ 23 ♡

MATH 200
April 16, 2026

1. Answer the questions about the function $f(x)$ graphed below.

(a) $\displaystyle\int_{-3}^{-1} f(x)\,dx = A_{up} - A_{down} = \frac{1}{2}1\cdot 1 - \frac{1}{2}1\cdot 1 = \boxed{0}$

(b) $\displaystyle\int_{4}^{2} f(x)\,dx = -\int_{2}^{4} f(x)\,dx = -\frac{1}{2}(2)(2) = \boxed{-2}$



(c) $\displaystyle\int_{-2}^{0} f(x)\,dx = \text{area} = \frac{1}{2}2\cdot 2 = \boxed{2}$

(d) Suppose $\displaystyle\int_{0}^{2} f(x)\,dx = 4.7$. Find $\displaystyle\int_{-2}^{4} f(x)\,dx$.

$$\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = 2 + 4.7 + 2 = \boxed{8.7}$$

(e) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} f\left(-3+\frac{2k}{n}\right)\frac{2}{n} = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \int_{-3}^{-1} f(x)\,dx = \boxed{0}$

$\Delta x = \frac{2}{n}$

$x_k = -3 + k\frac{2}{n} = -3 + k\Delta x$

$a = x_0 = -3 + 0\cdot\frac{2}{n} = -3$

$b = x_n = -3 + n\cdot\frac{2}{n} = -3 + 2 = -1$

2. Suppose for functions f and g we have: $\displaystyle\int_{1}^{4} f(x)\,dx = 1$, $\displaystyle\int_{4}^{6} f(x)\,dx = 2$, $\displaystyle\int_{1}^{6} g(x)\,dx = 3$.

Find $\displaystyle\int_{1}^{6} (2f(x) + g(x))\,dx$

$$= \int_1^6 2f(x)\,dx + \int_1^6 g(x)\,dx$$

$$= 2\int_1^6 f(x)\,dx + \int_1^6 g(x)\,dx$$

$$= 2\left(\int_1^4 f(x)\,dx + \int_4^6 f(x)\,dx\right) + \int_1^6 g(x)\,dx$$

$$= 2(1+2) + 3 = \boxed{9}$$

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QUIZ 23 ♣

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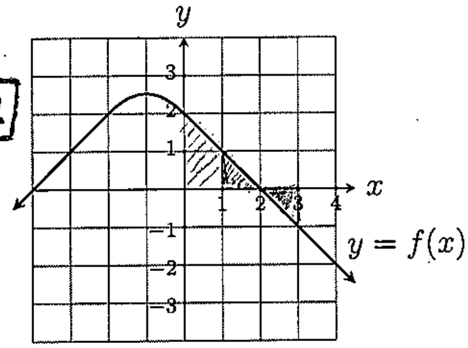
1. Answer the questions about the function $f(x)$ graphed below.

(a) $\int_1^3 f(x)\,dx = A_{up} - A_{down} = \frac{1}{2}1\cdot1 - \frac{1}{2}1\cdot1 = \boxed{0}$

(b) $\int_4^2 f(x)\,dx = -\int_2^4 f(x)dx = -(A_{up} - A_{down})$
$= -(0 - \frac{1}{2}2\cdot2) = \boxed{2}$

(c) $\int_0^1 f(x)\,dx = A_{up} - A_{down} = (\frac{3}{2} - 0) = \boxed{\frac{3}{2}}$



(d) Suppose $\int_{-2}^0 f(x)\,dx = 4.7$. Find $\int_{-2}^2 f(x)\,dx$.

$\int_{-2}^2 f(x)\,dx = \int_{-2}^0 f(x)\,dx + \int_0^2 f(x)\,dx = 4.7 + \frac{1}{2}2\cdot2 = \boxed{6.7}$

(e) $\lim_{n\to\infty} \sum_{k=1}^n f\left(\frac{k}{n}\right)\frac{1}{n} = \lim_{n\to\infty} \sum_{K=1}^n f(x_n)\Delta x = \int_0^1 f(x)dx = \boxed{\frac{3}{2}}$

$\Delta x = \frac{1}{n}$
$x_K = k\Delta x = \frac{k}{n}$
$a = x_0 = \frac{0}{n} = 0$
$b = x_n = n/n = 1$

2. Suppose for functions f and g we have: $\int_1^4 f(x)\,dx = 3$, $\int_4^6 f(x)\,dx = 2$, $\int_1^6 g(x)\,dx = 1$.

Find $\int_1^6 (5f(x) + g(x))\,dx$

$= \int_1^6 5f(x)dx + \int_1^6 g(x)\,dx$

$= 5\int_1^6 f(x)\,dx + \int_1^6 g(x)dx$

$= 5(\int_1^4 f(x)dx + \int_4^6 f(x)dx) + \int_1^6 g(x)dx$

$= 5(3 + 2) + 1 = 5\cdot5 + 1 = \boxed{26}$

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QUIZ 23 ◊

MATH 200
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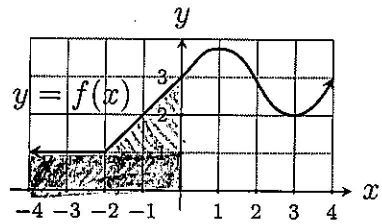
1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 17.8$, what is $\int_{0}^{4} f(x)\,dx$?



$$17.8 = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$17.8 = [\text{shaded area}] + \int_{0}^{4} f(x)\,dx$$

$$17.8 = 6 + \int_{0}^{4} f(x)\,dx \implies \int_{0}^{4} f(x)\,dx = \boxed{11.8}$$

2. Suppose f is a function for which $\int_2^5 f(x)dx = 4$ and $\int_2^8 f(x)dx = 9$. Find $\int_8^5 7f(x)dx$.

$$\int_2^8 f(x)\,dx = \int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx$$

$$9 = 4 + \int_5^8 f(x)\,dx \implies \boxed{\int_5^8 f(x)\,dx = 5}$$

Now $\int_8^5 7f(x)\,dx = 7\int_8^5 f(x)\,dx = -7\int_5^8 f(x)\,dx = -7 \cdot 5 = \boxed{-35}$

3. Write the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$ as a definite integral.

$x_k = \frac{\pi k}{n}$

k	$x_k = \frac{\pi k}{n}$
0	$x_0 = \frac{\pi \cdot 0}{n} = 0 \leftarrow a$
1	$x_1 = \frac{\pi \cdot 1}{n}$
⋮	
n	$x_n = \frac{\pi \cdot n}{n} = \pi \leftarrow b$

$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{n} = \frac{\pi}{n}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\sqrt{\frac{\pi k}{n}}\right)\frac{\pi}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sin(\sqrt{x_k})\Delta x$$

$$= \boxed{\int_0^{\pi} \sin(\sqrt{x})\,dx}$$

4. Write $\int_0^5 e^x\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$\Delta x = \frac{5-0}{n} = \frac{5}{n}$

$x_k = 0 + k\Delta x = \frac{5k}{n}$

$$\int_0^5 e^x\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} e^{5k/n} \cdot \frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{5e^{5k/n}}{n}$$

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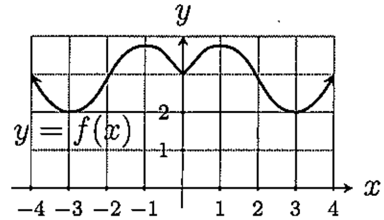
QUIZ 23 ♠

MATH 200
April 16, 2026

1. A function $f(x)$ is graphed below. If $\int_{-4}^{4} f(x)\,dx = 22.6$, what is $\int_{0}^{4} f(x)\,dx$?

By symmetry, $\int_{-4}^{0} f(x)\,dx = \int_{0}^{4} f(x)\,dx$, so



$\underline{22.6} = \int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx = \underline{2\int_{0}^{4} f(x)\,dx}$

Thus $\int_{0}^{4} f(x)\,dx = \dfrac{22.6}{2} = \boxed{11.3}$

2. Suppose f and g are functions for which $\int_0^5 f(x)\,dx = 3$, $\int_0^2 3g(x)\,dx = 12$, and $\int_2^5 g(x)\,dx = -1$. Find $\int_0^5 3f(x) - g(x)\,dx$.

$12 = \int_0^2 3g(x)\,dx \Rightarrow 12 = 3\int_0^2 g(x)\,dx \Rightarrow \int_0^2 g(x)\,dx = \frac{12}{3} = \boxed{4}$

Now, $\int_0^5 3f(x) - g(x)\,dx = \int_0^5 3f(x)\,dx - \int_0^5 g(x)\,dx = 3\int_0^5 f(x)\,dx - \int_0^5 g(x)\,dx$

$= 3\cdot 3 - \left(\int_0^2 g(x)\,dx + \int_2^5 g(x)\,dx\right) = 9 - (4 - 1) = \boxed{6}$

3. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{1+(7k/n)^2}\,\frac{7}{n}$ as a definite integral. $= \displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{1+x_k^2}\Delta x = \boxed{\int_0^7 \frac{1}{1+x^2}\,dx}$

K	7K/n
0	0 ← a
1	7·1/n
2	7·2/n
⋮	⋮
n	7·n/n = 7 ← b

$\Delta x = \dfrac{b-a}{n} = \dfrac{7-0}{n} = \dfrac{7}{n}$

Write $\int_3^4 \sqrt{x}\,dx$ as a limit of Riemann sums (such as in problem 3 above).

$\Delta x = \dfrac{4-3}{n} = \dfrac{1}{n}$

$x_k = 3 + k\Delta x$

$= 3 + \dfrac{k}{n}$

$\int_3^4 \sqrt{x}\,dx = \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{3+\frac{k}{n}}\cdot\frac{1}{n}}$