Name: Richard

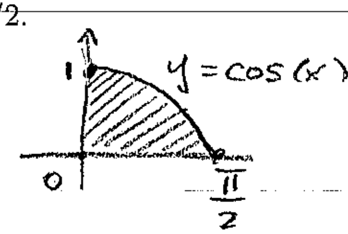
QUIZ 24

MATH 200
November 30, 2023

1. Find the area under the graph of $y = \cos(x)$ between $x = 0$ and $x = \pi/2$.

$$\int_0^{\pi/2} \cos(x)\,dx = \Big[\sin(x)\Big]_0^{\frac{\pi}{2}} = \sin\left(\tfrac{\pi}{2}\right) - \sin(0) = 1 - 0$$

$$= \boxed{1 \text{ square unit}}$$



2. $\displaystyle\int_0^4 (3x^2 + 2x)\,dx = \left[3\frac{x^3}{3} + 2\frac{x^2}{2}\right]_0^4 = \left[x^3 + x^2\right]_0^4 = (4^3 + 4^2) - (0^3 + 0^2)$

$$= 64 + 16 = \boxed{80}$$

3. $\displaystyle\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \int_1^2 (x^1 + x^{-2})\,dx = \left[\frac{x^2}{2} - \frac{1}{x}\right]_1^2 = \left(\frac{2^2}{2} - \frac{1}{2}\right) - \left(\frac{1^2}{2} - \frac{1}{1}\right)$

$$= 2 - \frac{1}{2} - \frac{1}{2} + 1 = 2 - 1 + 1 = \boxed{2}$$

4. Find the derivative of the function $\displaystyle F(x) = \int_x^0 \frac{e^t \sin(\pi t)}{t^5 + e^t}\,dt.$ $\displaystyle = -\int_0^x \frac{e^t \sin(\pi t)}{t^5 + e^t}\,dt$

By FTC I, $\boxed{F'(x) = -\dfrac{e^x \sin(\pi x)}{x^5 + e^x}}$

5. The graph of a function $f(x)$ is shown below.

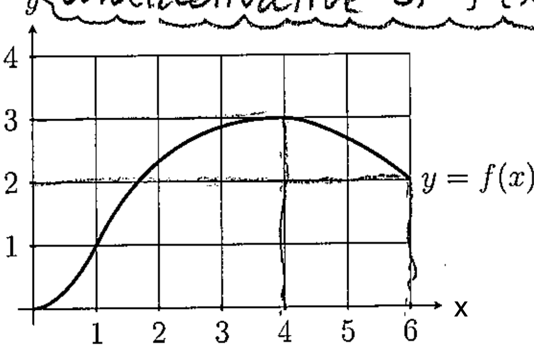
because $f(x)$ is an antiderivative of $f'(x)$

Find $\displaystyle\int_4^6 f'(t)\,dt.$ $= \Big[f(x)\Big]_4^6$

$$= f(6) - f(4)$$

$$= 2 - 3$$

$$= \boxed{-1}$$



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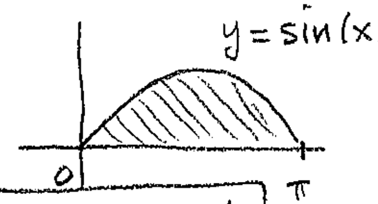
QUIZ 24

MATH 200
November 30, 2023

1. Find the area under the graph of $y = \sin(x)$ between $x = 0$ and $x = \pi$.

$$A = \int_0^{\pi} \sin(x)\,dx = \Big[-\cos(x)\Big]_0^{\pi} = -\cos(\pi) - (-\cos(0))$$
$$= -(-1) - (-1) = \boxed{2 \text{ square units}}$$



2. $\int_1^2 (2x - 3x^2 + 1)\,dx =$

$$= \left[2\frac{x^2}{2} - 3\frac{x^3}{3} + x\right]_1^2 = \left[x^2 - x^3 + x\right]_1^2 = (2^2 - 2^3 + 2) - (1^2 - 1^3 + 1)$$
$$= (4 - 8 + 2) - 1 = \boxed{-3}$$

3. $\int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx = \Big[\sin^{-1}(x)\Big]_0^{\sqrt{3}/2} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}(0)$

$$= \frac{\pi}{3} - 0 = \boxed{\frac{\pi}{3}}$$

4. Find the derivative of the function $F(x) = \int_x^{\pi} \frac{t^5 + e^t}{e^t \ln(t)}\,dt.$ $= -\int_{\pi}^{x} \frac{t^5 + e^t}{e^t \ln(t)}\,dt$

By FTC I, $\boxed{F'(x) = -\frac{x^5 + e^x}{e^x \ln(x)}}$

because $f(x)$ is an antiderivative of $f'(x)$!

5. The graph of a function $f(x)$ is shown below.

Find $\int_1^4 f'(t)\,dt.$ $= \Big[f(x)\Big]_1^4$

$$= f(4) - f(1)$$
$$= 1 - 3$$
$$= \boxed{-2}$$