Name: ______________________________ **QUIZ 24** **MATH 200**
April 26, 2022

1. Find the area under the graph of $y = x^2 + 2$ between $x = -1$ and $x = 1$.

$$\int_{-1}^{1} \left(x^2+2\right)\, dx = \left[\frac{x^3}{3} + 2x\right]_{-1}^{1} = \left(\frac{1^3}{3} + 2\cdot 1\right) - \left(\frac{(-1)^3}{3} + 2(-1)\right) = \frac{2}{3} + 4 = \boxed{\frac{14}{3} \text{ square units}}$$

2. $$\int_0^4 \left(\sqrt{x} + 2x\right)\, dx = \int_0^4 \left(x^{1/2} + 2x\right)\, dx = \left[\frac{x^{1/2+1}}{1/2+1} + x^2\right]_0^4 = \left[\frac{2x^{3/2}}{3} + x^2\right]_0^4 = \left[\frac{2\sqrt{x}^3}{3} + x^2\right]_0^4$$

$$= \left(\frac{2\sqrt{4}^3}{3} + 4^2\right) - \left(\frac{2\sqrt{0}^3}{3} + 0^2\right) = \frac{16}{3} + 16 = \frac{16}{3} + \frac{48}{3} = \boxed{\frac{64}{3}}$$

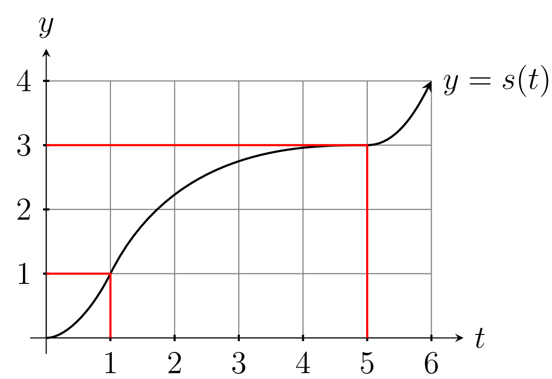
3. $$\int_1^{\sqrt{3}} \frac{1}{1+x^2}\, dx = \left[\tan^{-1}(x)\right]_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \boxed{\frac{\pi}{12}}$$

4. Find the derivative of the function $F(x) = \displaystyle\int_0^x \frac{e^t \sin(\pi t)}{t^5 + e^t}\, dt$.

By the Fundamental Theorem of Calculus, Part I, $\boxed{F'(x) = \dfrac{e^x \sin(\pi x)}{x^5 + e^x}}$

5. An object moving on a line has position $s(t)$ and velocity $v(t)$ at time t.
The position function $s(t)$ is graphed below.

Find $\displaystyle\int_1^5 v(t)\, dt$.



Because $\displaystyle\int v(t)\, dt = s(t)$, we have $\displaystyle\int_1^5 v(t)\, dt = \big[s(t)\big]_1^5 = s(5) - s(1) = 3 - 1 = \boxed{2}$

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1. Find the area under the graph of $y = \sqrt{x}$ between $x = 1$ and $x = 4$.

$$\int_1^4 \sqrt{x}\, dx = \int_1^4 x^{1/2}\, dx = \left[\frac{x^{1/2+1}}{1/2+1}\right]_1^4 = \left[\frac{x^{3/2}}{3/2}\right]_1^4 = \left[\frac{2x^{3/2}}{3}\right]_1^4 = \left[\frac{2\sqrt{x}^3}{3}\right]_1^4 = \frac{2\sqrt{4}^3}{3} - \frac{2\sqrt{1}^3}{3}$$

$$= \frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3} \text{ square units}}$$

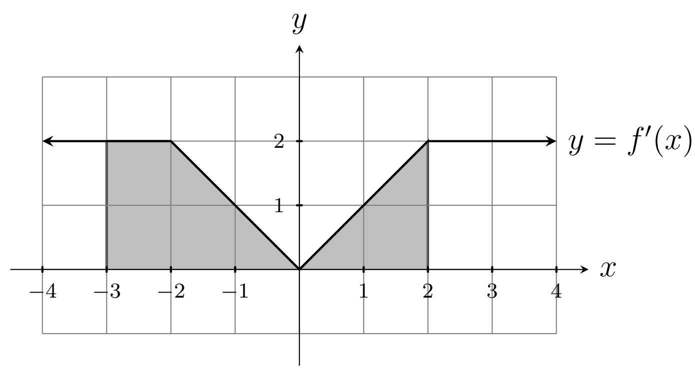
2. $$\int_0^1 \left(x^2 + 2x + 1\right) dx = \left[\frac{x^3}{3} + x^2 + x\right]_0^1 = \left(\frac{1^3}{3} + 1^2 + 1\right) - \left(\frac{0^3}{3} + 0^2 + 0\right) = \frac{1}{3} + \frac{3}{3} + \frac{3}{3} = \boxed{\frac{7}{3}}$$

3. $$\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\, dx = \left[\sin^{-1}(x)\right]_{-1}^1 = \sin^{-1}(1) - \sin^{-1}(-1) = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \boxed{\pi}$$

4. Find the derivative of the function $F(x) = \displaystyle\int_\pi^x \frac{t^5 + e^t}{e^t \sin(\pi t)}\, dt$.

By the Fundamental Theorem of Calculus, Part I, $\boxed{F'(x) = \dfrac{x^5 + e^x}{e^x \sin(\pi x)}}$

5. The derivative $f'(x)$ of a function $f(x)$ is graphed below. Suppose $f(2) = 3$. Find $f(-3)$.



By the Fundamental Theorem of Calculus, Part II, $\displaystyle\int_{-3}^2 f'(x)\, dx = f(2) - f(-3) = 3 - f(-3)$.

That is, $\displaystyle\int_{-3}^2 f'(x)\, dx = 3 - f(-3)$.

The shaded area is $\displaystyle\int_{-3}^2 f'(x)\, dx = 6$. Thus the above equation is $6 = 3 - f(-3)$, so $\boxed{f(-3) = -3}$