Name: Richard

QUIZ 25 ♡

MATH 200
December 12, 2022

1. $\int \frac{e^x}{\sqrt{e^x}}\,dx = \int (e^x)^{-\frac{1}{2}} e^x dx = \int u^{-1/2} du$

$u = e^x$
$\frac{du}{dx} = e^x$
$du = e^x dx$

$= \frac{u^{1/2}}{1/2} + C = 2\sqrt{u} + C$

$= \boxed{2\sqrt{e^x} + C}$

2. $\int \sin^2(\pi x)\cos(\pi x)\,dx = \int u^2 \frac{1}{\pi} du = \frac{1}{\pi}\int u^2 du = \frac{1}{\pi}\frac{u^3}{3} + C$

$u = \sin(\pi x)$
$\frac{du}{dx} = \cos(\pi x)\pi$
$du = \pi\cos(\pi x)dx \rightarrow \frac{1}{\pi}du = \cos(\pi x)dx$

$= \boxed{\frac{\sin^3(\pi x)}{3\pi} + C}$

3. $\int_0^{\pi/2} \frac{\cos(x)}{\sin(x)+5}\,dx = \int_0^{\pi/2} \frac{1}{\sin(x)+5}\cos(x)\,dx$

$u = \sin(x) + 5$
$\frac{du}{dx} = \cos(x)$
$du = \cos(x)dx$

$= \int_{\sin(0)+5}^{\sin(\pi/2)+5} \frac{1}{u} du = \int_5^6 \frac{1}{u} du$

$= \left[\ln(u)\right]_5^6 = \ln(6) - \ln(5)$

$= \boxed{\ln\left(6/5\right)}$

4. Find the area under the graph of $\sec^2(2x)$ between 0 and $\pi/8$.

$\int_0^{\pi/8} \sec^2(2x)\,dx = \int_{2\cdot 0}^{2\cdot\pi/8} \sec^2(u)\frac{1}{2}du = \frac{1}{2}\int_0^{\pi/4} \sec^2(u)\,du$

$u = 2x$
$du = 2dx \rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\left[\tan(u)\right]_0^{\pi/4} = \frac{1}{2}\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right)$

$= \frac{1}{2}(1-0) = \boxed{\frac{1}{2}\text{ sq unit}}$

Name: Richard

QUIZ 25 ♣

MATH 200
December 12, 2022

1. $\displaystyle\int 12x^2\sqrt{4x^3+15}\,dx = \int (4x^3+15)^{1/2}\,12x^2\,dx = \int u^{1/2}\,du$

$u = 4x^3+15$

$\frac{du}{dx} = 12x^2$

$du = 12x^2\,dx$

$= \dfrac{u^{3/2}}{3/2} + C = \dfrac{2\sqrt{u}^3}{3} + C$

$= \boxed{\dfrac{2\sqrt{4x^3+15}^3}{3} + C}$

2. $\displaystyle\int \frac{2x^9-e^x}{x^{10}-5e^x}\,dx = \int \frac{1}{x^{10}-5e^x}(2x^9-e^x)\,dx = \int \frac{1}{u}\frac{1}{5}\,du$

$u = x^{10}-5e^x$

$\frac{du}{dx} = 10x^9 - 5e^x$

$du = (10x^9-5e^x)\,dx \rightarrow \frac{1}{5}du = (2x^9-e^x)\,dx$

$= \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\ln|u| + C = \boxed{\frac{1}{5}\ln|x^{10}-5e^x| + C}$

3. $\displaystyle\int_0^3 (x^2-4x+1)^3(2x-4)\,dx = \int_{0^2-4\cdot 0+1}^{3^2-4\cdot 3+1} u^3\,du = \int_1^{-2} u^3\,du$

$u = x^2-4x+1$

$\frac{du}{dx} = 2x-4$

$du = (2x-4)\,dx$

$= \left[\frac{u^4}{4}\right]_1^{-2} = \frac{(-2)^4}{4} - \frac{1^4}{4} = \boxed{\frac{15}{4}}$

4. Find the area under the graph of $x\sin(x^2)$ between 0 and $\sqrt{\pi/6}$.

$A = \displaystyle\int_0^{\sqrt{\pi/6}} \sin(x^2)\,x\,dx = \int_{0^2}^{\sqrt{\pi/6}^2} \sin(u)\,\frac{1}{2}du = \frac{1}{2}\int_0^{\frac{\pi}{6}} \sin(u)\,du$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\left[-\cos(u)\right]_0^{\frac{\pi}{6}} = \frac{1}{2}\left(-\cos\left(\frac{\pi}{6}\right) - (-\cos(0))\right)$

$= \frac{1}{2}\left(-\frac{\sqrt{3}}{2}+1\right) = \boxed{\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right) \text{ sq units}}$

Name: Richard

Quiz 25 ♢

MATH 200
December 12, 2022

1. $\int \sqrt{\sin(x)}\cos(x)\,dx = \int (\sin(x))^{\frac{1}{2}} \cos(x)\,dx$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= \int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{u^{3/2}}{3/2} + C = \frac{2}{3}\sqrt{u}^{\,3} + C = \boxed{\frac{2}{3}\sqrt{\sin(x)}^{\,3} + C}$

2. $\int \frac{\sin(2x)}{\cos^5(2x)}\,dx = \int (\cos(2x))^{-5}\sin(2x)\,dx = \int u^{-5}\left(-\frac{1}{2}\,du\right)$

$u = \cos(2x)$

$\frac{du}{dx} = -\sin(2x)\cdot 2$

$du = -\sin(2x)\,2\,dx \rightsquigarrow \boxed{-\frac{1}{2}\,du = \sin(2x)\,dx}$

$= -\frac{1}{2}\int u^{-5}\,du = -\frac{1}{2}\frac{u^{-4}}{-4} + C = \frac{1}{8u^4} + C$

$= \boxed{\frac{1}{8\cos^4(2x)} + C}$

3. $\int_0^{\sqrt{\pi/4}} \sec^2(x^2)\,x\,dx = \int_{0^2}^{\sqrt{\pi/4}^{\,2}} \sec^2(u)\,\frac{1}{2}\,du = \frac{1}{2}\int_0^{\pi/4} \sec^2(u)\,du$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$= \frac{1}{2}\Big[\tan(u)\Big]_0^{\frac{\pi}{4}} = \frac{1}{2}\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right)$

$= \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$

4. Find the area under the graph of $\dfrac{3}{3x+7}$ between -2 and 1.

$A = \int_{-2}^{1} \frac{3}{3x+7}\,dx = \int_{-2}^{1} \frac{1}{3x+7}\,3\,dx = \int_{3(-2)+7}^{3(1)+7} \frac{1}{u}\,du = \int_1^{10} \frac{1}{u}\,du$

$u = 3x+7$

$\frac{du}{dx} = 3 \rightsquigarrow \boxed{du = 3\,dx}$

$= \Big[\ln|u|\Big]_1^{10} = \ln|10| - \ln|1| =$

$\boxed{\ln|10| \text{ sq. units}}$

Name: Richard

QUIZ 25 ♠

MATH 200
December 12, 2022

1. $\int \frac{\sec^2(-1/x)}{x^2}\,dx = \int \sec^2\left(-\frac{1}{x}\right)\frac{1}{x^2}\,dx = \int \sec^2(u)\,du$

$u = -\frac{1}{x}$

$\frac{du}{dx} = \frac{1}{x^2}$

$du = \frac{1}{x^2}\,dx$

$= \tan(u) + C = \boxed{\tan(-1/x) + C}$

2. $\int 2e^{-x}\,dx = 2\int e^{-x}\,dx = 2\int e^{u}(-du) = -2\int e^{u}\,du$

$u = -x$

$\frac{du}{dx} = -1$

$du = -dx \rightsquigarrow dx = -du$

$= -2e^{u} + C = \boxed{-2e^{-x} + C}$

3. $\int_{-1}^{0} \frac{x}{1+x^2}\,dx = \int_{-1}^{0} \frac{1}{1+x^2}\,x\,dx = \int_{1+(-1)^2}^{1+0^2} \frac{1}{u}\,\frac{1}{2}\,du = \frac{1}{2}\int_{2}^{1} \frac{1}{u}\,du$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\Big[\ln|u|\Big]_{2}^{1} = \frac{1}{2}\left(\ln|1| - \ln|2|\right)$

$= \frac{1}{2}\left(0 - \ln|2|\right) = \boxed{-\frac{\ln|2|}{2}}$

4. Find the area under the graph of $\frac{5}{(5x+1)^2}$ between 0 and 1.

$A = \int_0^1 \frac{5}{(5x+1)^2}\,dx = \int_0^1 (5x+1)^{-2}\,5\,dx = \int_{5\cdot 0+1}^{5\cdot 1+1} u^{-2}\,du$

$u = 5x+1$

$\frac{du}{dx} = 5$

$du = 5\,dx$

$= \int_1^6 u^{-2}\,du = \Big[-u^{-1}\Big]_1^6 = \Big[-\frac{1}{u}\Big]_1^6 = -\frac{1}{6} - \left(-\frac{1}{1}\right)$

$= \boxed{5/6 \text{ sq units}}$