



$$1. \int \cos(1+e^x) e^x dx = \int \cos(u) du = \sin(u) + C = \boxed{\sin(1+e^x) + C}$$

$$\begin{aligned} u &= 1+e^x \\ \frac{du}{dx} &= 0+e^x \\ du &= e^x dx \end{aligned}$$

$$2. \int e^{1-\cos(x)} \sin(x) dx = \int e^u du = e^u + C = \boxed{e^{1-\cos(x)} + C}$$

$$\begin{aligned} u &= 1-\cos(x) \\ \frac{du}{dx} &= 0+\sin(x) \\ du &= \sin(x) dx \end{aligned}$$

$$3. \int \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) \frac{1}{x^2} dx = \int \sec(u) \tan(u) (-1) du = -\int \sec(u) \tan(u) du$$

$$\begin{aligned} u &= \frac{1}{x} \\ \frac{du}{dx} &= -\frac{1}{x^2} \\ du &= -\frac{1}{x^2} dx \rightarrow -du = \frac{1}{x^2} dx \end{aligned}$$

$$= -\sec(u) + C$$

$$= \boxed{-\sec\left(\frac{1}{x}\right) + C}$$

$$4. \int_0^{\sqrt{\pi}} x \sin(x^2) dx = \int_0^{\sqrt{\pi/2}} \sin(x^2) x dx = \int_0^{\left(\frac{\sqrt{\pi}}{2}\right)^2} \sin(u) \frac{1}{2} du$$

$$\begin{aligned} u &= x^2 \\ \frac{du}{dx} &= 2x \\ du &= 2x dx \\ \frac{1}{2} du &= x dx \end{aligned}$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin(u) du = \frac{1}{2} \left[-\cos(u) \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \left(-\cos\left(\frac{\pi}{4}\right) - (-\cos(0)) \right) = \boxed{\frac{1}{2} \left(-\frac{\sqrt{2}}{2} + 1 \right)}$$