



$$1. \int \frac{(\ln|x|)^3}{x} dx = \int (\ln|x|)^3 \frac{1}{x} dx = \int u^3 du = \frac{u^4}{4} + C$$

$$= \frac{(\ln|x|)^4}{4} + C$$

$$\begin{aligned} u &= \ln|x| \\ \frac{du}{dx} &= \frac{1}{x} \\ du &= \frac{1}{x} dx \end{aligned}$$

$$2. \int e^x \sqrt{e^x+1} dx = \int (e^x+1)^{\frac{1}{2}} e^x dx = \int u^{\frac{1}{2}} du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{u}^3}{3} + C = \frac{2\sqrt{e^x+1}^3}{3} + C$$

$$\begin{aligned} u &= e^x+1 \\ \frac{du}{dx} &= e^x \\ du &= e^x dx \end{aligned}$$

$$3. \int \frac{x+e^x}{x^2+2e^x} dx = \int \frac{1}{x^2+2e^x} \frac{1}{2}(x+e^x) dx = \int \frac{1}{u} \frac{1}{2} du = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2+2e^x| + C$$

$$4. \int_0^{\frac{\sqrt{\pi}}{2}} x \sec^2(x^2) dx = \int_0^{\left(\frac{\sqrt{\pi}}{2}\right)^2} \sec^2(u) \frac{1}{2} du$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} \sec^2(u) du = \frac{1}{2} \left[\tan(u) \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \left(\tan\left(\frac{\pi}{4}\right) - \tan(0) \right) = \frac{1}{2} (1-0) = \frac{1}{2}$$

$$\begin{aligned} u &= x^2 \\ du &= 2x dx \\ \frac{1}{2} du &= x dx \end{aligned}$$



$$1. \int \frac{\sec^2(\ln|x|)}{x} dx = \int \sec^2(\ln|x|) \frac{1}{x} dx = \int \sec^2(u) du$$

$$= \tan(u) + C = \boxed{\tan(\ln|x|) + C}$$

$$\begin{cases} u = \ln|x| \\ \frac{du}{dx} = \frac{1}{x} \\ du = \frac{1}{x} dx \end{cases}$$

$$2. \int \cos^5(x) \sin(x) dx = \int (\cos(x))^5 \sin(x) dx = \int u^5 (-du) = -\int u^5 du$$

$$= -\frac{u^6}{6} + C = -\frac{(\cos(x))^6}{6} + C$$

$$= \boxed{-\frac{1}{6} \cos^6(x) + C}$$

$$\begin{cases} u = \cos(x) \\ \frac{du}{dx} = -\sin(x) \\ -du = \sin(x) dx \end{cases}$$

$$3. \int \frac{6x^2 + 6x}{2x^3 + 3x^2 + 6} dx = \int \frac{1}{2x^3 + 3x^2 + 6} (6x^2 + 6x) dx = \int \frac{1}{u} du$$

$$= \ln|u| + C = \boxed{\ln|2x^3 + 3x^2 + 6| + C}$$

$$\begin{cases} u = 2x^3 + 3x^2 + 6 \\ \frac{du}{dx} = 6x^2 + 6x \\ du = (6x^2 + 6x) dx \end{cases}$$

$$4. \int_0^1 x e^{x^2-1} dx = \int_0^1 e^{x^2-1} x dx = \int_{0^2-1}^{1^2-1} e^u \frac{1}{2} du = \frac{1}{2} \int_{-1}^0 e^u du$$

$$= \frac{1}{2} [e^u]_{-1}^0 = \frac{1}{2} (e^0 - e^{-1})$$

$$= \frac{1}{2} \left(1 - \frac{1}{e}\right) = \frac{1}{2} - \frac{1}{2e} = \boxed{\frac{e-1}{2e}}$$

$$\begin{cases} u = x^2 - 1 \\ \frac{du}{dx} = 2x \\ du = 2x dx \\ \frac{1}{2} du = x dx \end{cases}$$