

Name: _____

Directions: Find the limits. Show all steps. Simplify your answer.

$$1. \quad \lim_{x \rightarrow 0} \frac{5x^2 + 3x}{3x} = \lim_{x \rightarrow 0} \frac{x(5x + 3)}{3x} = \lim_{x \rightarrow 0} \frac{5x + 3}{3} = \frac{5 \cdot 0 + 3}{3} = \boxed{1}$$

$$2. \quad \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2} \cdot \frac{\sqrt{x^2 + 12} + 4}{\sqrt{x^2 + 12} + 4} =$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12}^2 + 4\sqrt{x^2 + 12} - 4\sqrt{x^2 + 12} - 4^2}{(x - 2)(\sqrt{x^2 + 12} + 4)} = \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12}^2 - 4^2}{(x - 2)(\sqrt{x^2 + 12} + 4)} =$$

$$\lim_{x \rightarrow 2} \frac{x^2 + 12 - 16}{(x - 2)(\sqrt{x^2 + 12} + 4)} = \lim_{x \rightarrow 2} \frac{x^2 - 4}{(x - 2)(\sqrt{x^2 + 12} + 4)} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{(x - 2)(\sqrt{x^2 + 12} + 4)} =$$

$$\lim_{x \rightarrow 2} \frac{x + 2}{\sqrt{x^2 + 12} + 4} = \frac{2 + 2}{\sqrt{2^2 + 12} + 4} = \frac{4}{\sqrt{16} + 4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$3. \quad \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - 1}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - 1}{h} \cdot \frac{1+h}{1+h} = \lim_{h \rightarrow 0} \frac{1 - (1+h)}{h(1+h)} = \lim_{h \rightarrow 0} \frac{1 - 1 - h}{h(1+h)} =$$

$$\lim_{h \rightarrow 0} \frac{-h}{h(1+h)} = \lim_{h \rightarrow 0} \frac{-1}{1+h} = \frac{-1}{1+0} = \boxed{-1}$$

$$4. \quad \lim_{x \rightarrow 2^+} \frac{4 - x^2}{|2 - x|} = \lim_{x \rightarrow 2^+} \frac{(2 - x)(2 + x)}{-(2 - x)} = \lim_{x \rightarrow 2^+} \frac{2 + x}{-1} = \frac{2 + 2}{-1} = \boxed{-4}$$

Note: When $2 < x$ (as it is when x approaches 2 from the right), the expression $2 - x$ is *negative*, so $|2 - x| = -(2 - x)$

Directions: Find the limits. Show all steps. Simplify your answer.

$$1. \quad \lim_{x \rightarrow 0} \frac{5x^2 + x^3}{5x^2} = \lim_{x \rightarrow 0} \frac{x^2(5 + x)}{5x^2} = \lim_{x \rightarrow 0} \frac{5 + x}{5} = \frac{5 + 0}{5} = \boxed{1}$$

$$2. \quad \lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3} = \lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3} \cdot \frac{9x^2}{9x^2} = \lim_{x \rightarrow 3} \frac{9 - x^2}{(x - 3)9x^2} = \lim_{x \rightarrow 3} \frac{(3 - x)(3 + x)}{(x - 3)9x^2} =$$

$$\lim_{x \rightarrow 3} \frac{-(x - 3)(3 + x)}{(x - 3)9x^2} = \lim_{x \rightarrow 3} \frac{-(3 + x)}{9x^2} = \frac{-(3 + 3)}{9 \cdot 3^2} = \frac{-6}{81} = \boxed{-\frac{2}{27}}$$

$$3. \quad \lim_{h \rightarrow 0} \frac{\sqrt{5 + h} - \sqrt{5}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{5 + h} - \sqrt{5}}{h} \cdot \frac{\sqrt{5 + h} + \sqrt{5}}{\sqrt{5 + h} + \sqrt{5}} =$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{5 + h}^2 + \sqrt{5 + h}\sqrt{5} - \sqrt{5}\sqrt{5 + h} - \sqrt{5}^2}{h(\sqrt{5 + h} + \sqrt{5})} = \lim_{h \rightarrow 0} \frac{\sqrt{5 + h}^2 - \sqrt{5}^2}{h(\sqrt{5 + h} + \sqrt{5})} =$$

$$\lim_{h \rightarrow 0} \frac{5 + h - 5}{h(\sqrt{5 + h} + \sqrt{5})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{5 + h} + \sqrt{5})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{5 + h} + \sqrt{5}} = \frac{1}{\sqrt{5} + \sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$$

$$4. \quad \lim_{x \rightarrow 1^+} \frac{|1 - x|}{6x - 6x^2} = \lim_{x \rightarrow 1^+} \frac{-(1 - x)}{6x - 6x^2} = \lim_{x \rightarrow 1^+} \frac{-(1 - x)}{6x(1 - x)} = \lim_{x \rightarrow 1^+} \frac{-1}{6x} = \frac{-1}{6 \cdot 1} = \boxed{-\frac{1}{6}}$$

Note: When $1 < x$ (as it is when x approaches 1 from the right), the expression $1 - x$ is *negative*, so $|1 - x| = -(1 - x)$