Name: Richard

QUIZ 3 ♡

MATH 200
September 8, 2021

1. $\lim_{x\to 0} \frac{\tan(x)}{3x} = \lim_{x\to 0} \frac{1}{3}\,\frac{\sin(x)}{x}\,\frac{1}{\cos(x)} = \frac{1}{3}\cdot 1\cdot \frac{1}{\cos(0)} = \frac{1}{3}\cdot 1\cdot 1 = \boxed{\frac{1}{3}}$

2. $\lim_{x\to 2} \ln\left(\frac{x^2-3x+2}{x-2}\right) = \ln\left(\lim_{x\to 2} \frac{x^2-3x+2}{x-2}\right)$

$= \ln\left(\lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}\right)$

$= \ln\left(\lim_{x\to 2} (x-1)\right)$

$= \ln(2-1) = \ln(1) = \boxed{0}$

3. State the intervals on which the function $f(x) = \frac{\sqrt{5-x}}{e^x - 1}$ is continuous.

Because it is built from continuous functions f is continuous on its domain, which is

$(-\infty\ 0) \cup (0, 5]$

(Because domain of $\sqrt{5-x}$ is $(-\infty, 5]$, but $x=0$ makes denominator e^x-1 zero.)

4. Draw the graph of **one** function f, with domain $(-4, 4)$, meeting **all** of the following conditions.

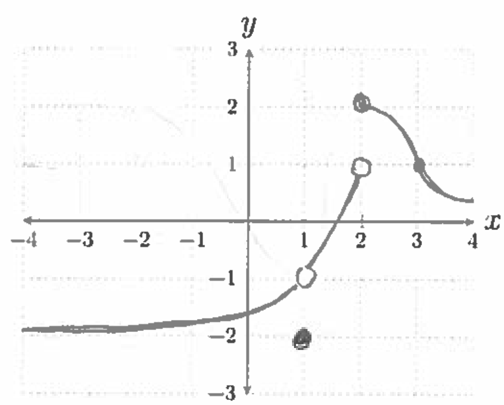
(a) f is continuous at all x except $x = 1$ and $x = 2$.

(b) $f(3) = 1$

(c) $\lim_{x\to 1} f(x) = -1$

(d) $\lim_{x\to 2^-} f(x) = 1$

(e) $\lim_{x\to 2^+} f(x) = 2$



Name: Richard

Quiz 3 ◊

MATH 200
September 8, 2021

1. $\lim\limits_{x\to 1} \dfrac{\sin(x^2-1)}{x^2-1} = \boxed{1}$ because $x^2-1 \to 0$ as $x \to 1$.

2. $\lim\limits_{x\to 0} \sin\left(\dfrac{\pi x}{6x-6x^2}\right) = \sin\left(\lim\limits_{x\to 0} \dfrac{\pi x}{6x-6x^2}\right)$

$$= \sin\left(\lim_{x\to 0} \frac{\cancel{x}\cdot\pi}{\cancel{x}(6-6x)}\right)$$

$$= \sin\left(\lim_{x\to 0} \frac{\pi}{6-6x}\right)$$

$$= \sin\left(\frac{\pi}{6-6\cdot 0}\right) = \sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$$

3. State the intervals on which the function $f(x) = \dfrac{1}{\ln(x)}$ is continuous.

The function $\ln(x)$ is continuous on its domain, which is $(0,\infty)$. Therefore $\dfrac{1}{\ln(x)}$ will be continuous on its domain which is

$$\boxed{(0,1)\cup(1,\infty)}$$

(note: $\ln(x)=0$ when $x=1$.)

4. Draw the graph of **one** function f, with domain $(-4,4)$, meeting **all** of the following conditions.

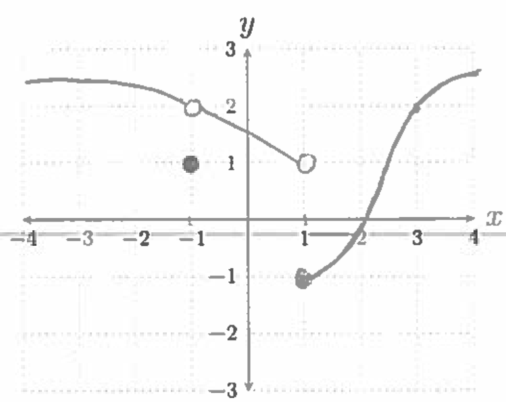
(a) f is continuous at all x except $x=-1$ and $x=1$.

(b) $f(3)=2$

(c) $\lim\limits_{x\to -1} f(x) = 2$

(d) $\lim\limits_{x\to 1^-} f(x) = 1$

(e) $\lim\limits_{x\to 1^+} f(x) = -1$



Name: Richard

QUIZ 3 ♣

MATH 200
September 8, 2021

1. $\lim_{x\to 0} \frac{\sin(x)+x}{x} = \lim_{x\to 0}\left(\frac{\sin(x)}{x} + \frac{x}{x}\right) = 1+1 = \boxed{2}$

2. $\lim_{x\to 3} \log_2\left(\frac{x^2+2x-15}{x-3}\right) = \log_2\left(\lim_{x\to 3}\frac{x^2+2x-15}{x-3}\right)$

$= \log_2\left(\lim_{x\to 3}\frac{\cancel{(x-3)}(x+5)}{\cancel{x-3}}\right)$

$= \log_2\left(\lim_{x\to 3}(x+5)\right) = \log_2(3+5)$

$= \log_2(8) = \boxed{3}$

3. State the intervals on which the function $f(x) = \sqrt{x^2+5}$ is continuous.

The function x^2+5 is continuous on $(-\infty, \infty)$ so $\sqrt{x^2-5}$ will be continuous wherever $x^2+5 \geq 0$ which is all real numbers. Therefore the function f is $\boxed{\text{continuous on } (-\infty, \infty) = \mathbb{R}}$

4. Draw the graph of **one** function f, with domain $(-4, 4)$, meeting **all** of the following conditions.

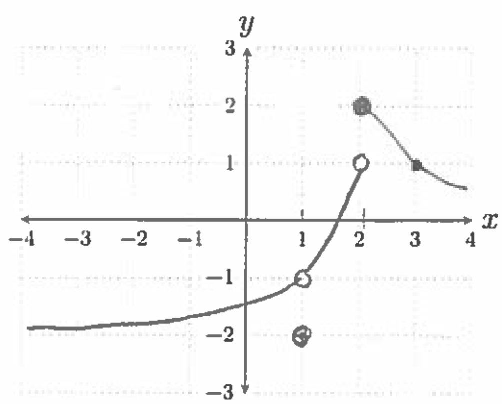
(a) f is continuous at all x except at $x = 1$ and $x = 2$.

(b) $f(3) = 1$

(c) $\lim_{x\to 1} f(x) = -1$

(d) $\lim_{x\to 2^-} f(x) = 1$

(e) $\lim_{x\to 2^+} f(x) = 2$



Name: Richard

QUIZ 3 ♠

MATH 200
September 8, 2021

1. $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{2x}$ $= \dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{\sin(3x)}{x} = \dfrac{3}{2} \lim\limits_{x\to 0} \dfrac{\sin(3x)}{3x} = \dfrac{3}{2}\cdot 1 = \boxed{\dfrac{3}{2}}$

2. $\lim\limits_{x\to\pi/6} \ln\left(\sin(x)+\dfrac{1}{2}\right)$ $= \ln\left(\lim\limits_{x\to\frac{\pi}{6}}\left(\sin(x)+\dfrac{1}{2}\right)\right)$

$= \ln\left(\sin\left(\dfrac{\pi}{6}\right)+\dfrac{1}{2}\right)$

$= \ln\left(\dfrac{1}{2}+\dfrac{1}{2}\right) = \ln(1) = \boxed{0}$

3. State the intervals on which the function $f(x) = \dfrac{\sqrt{x+6}}{x^2-3x+2}$ is continuous.

$f(x) = \dfrac{\sqrt{x+6}}{(x-1)(x-2)}$ will be continuous on its domain because it is built up from continuous functions. Its domain is $\boxed{[-6,1)\cup(1,2)\cup(2,\infty)}$

4. Draw the graph of **one** function f, with domain $(-4,4)$, meeting **all** of the following conditions.

(a) f is continuous at all x except $x=-1$ and $x=1$.

(b) $f(3)=2$

(c) $\lim\limits_{x\to -1} f(x) = 2$

(d) $\lim\limits_{x\to 1^-} f(x) = 1$

(e) $\lim\limits_{x\to 1^+} f(x) = -1$