Name: Richard

QUIZ 3

MATH 200

August 29, 2024

1. $\lim_{x\to 1} \frac{\sin(x-1)}{3x-3} = \lim_{x\to 1} \frac{1}{3} \frac{\sin(x-1)}{x-1} = \frac{1}{3} \lim_{x\to 1} \frac{\sin(x-1)}{x-1} = \frac{1}{3}\cdot 1 = \boxed{\frac{1}{3}}$

2. $\lim_{x\to 0} \cos\left(\frac{\pi x}{6x-6x^2}\right) = \cos\left(\lim_{x\to 0} \frac{\pi x}{6x-6x^2}\right) = \cos\left(\frac{\pi}{6} \lim_{x\to 0} \frac{x}{x-x^2}\right)$

$= \cos\left(\frac{\pi}{6} \lim_{x\to 0} \frac{x}{x(1-x)}\right) = \cos\left(\frac{\pi}{6} \lim_{x\to 0} \frac{1}{1-x}\right)$

$= \cos\left(\frac{\pi}{6} \frac{1}{1-0}\right) = \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

3. State the intervals on which the function $f(x) = \frac{1}{1-\ln(x)}$ is continuous.

Because the numerator and denominator are continuous on their domains, this function is also continuous on its domain, which is $\boxed{(0,e)\cup(e,\infty)}$

(Note the domain of $\ln(x)$ is $(0,\infty)$ but $x=e$ makes the denominator 0, so we must eliminate e from $(0,\infty)$)

4. Draw the graph of **one** function f, with domain $[-4,4]$, meeting **all** of the following conditions.

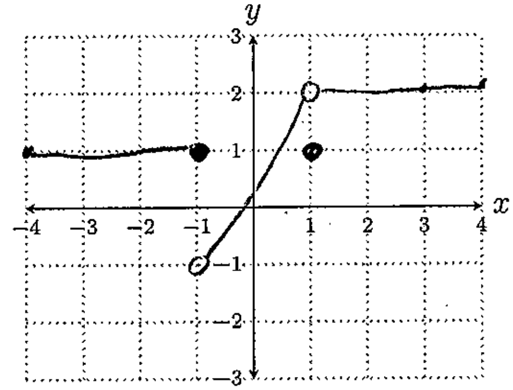
(a) f is continuous at all x except $x=-1$ and $x=1$.

(b) $f(3) = 2$

(c) $\lim_{x\to 1} f(x) = 2$

(d) $\lim_{x\to -1^-} f(x) = 1$

(e) $\lim_{x\to -1^+} f(x) = -1$



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QUIZ 3

MATH 200
August 29, 2024

1. $\lim_{x\to 0} \frac{7\sin(x)}{3x} = \frac{7}{3} \lim_{x\to 0} \frac{\sin(x)}{x} = \frac{7}{3}\cdot 1 = \boxed{\frac{7}{3}}$

2. $\lim_{x\to 5} \log_3\left(\frac{x^2 - x - 20}{x-5}\right) = \log_3\left(\lim_{x\to 5} \frac{x^2 - x - 20}{x-5}\right)$

$= \log_3\left(\lim_{x\to 5} \frac{(x+4)\cancel{(x-5)}}{\cancel{x-5}}\right) = \log_3\left(\lim_{x\to 5} (x+4)\right)$

$= \log_3(9) = \boxed{2}$

3. State the intervals on which the function $f(x) = \frac{\sqrt{x+2}}{e^x - e}$ is continuous.

Because numerator and denominator are continuous on their domains this function is continuous on its domain, which is $\boxed{[-2,1) \cup (1,\infty)}$

(Note: $\sqrt{x+2}$ has domain $[-2,\infty)$, however $x=1$ makes the denominator 0, so we have to eliminate $x=1$.)

4. Draw the graph of **one** function f, with domain $[-4,4]$, meeting **all** of the following conditions.

(a) f is continuous at all x except $x=1$ and $x=2$.

(b) $f(3) = -2$

(c) $\lim_{x\to 2} f(x) = -1$

(d) $\lim_{x\to 1^-} f(x) = 1$

(e) $\lim_{x\to 1^+} f(x) = 2$