Name: Richard

Quiz 3

MATH 200
August 28, 2025

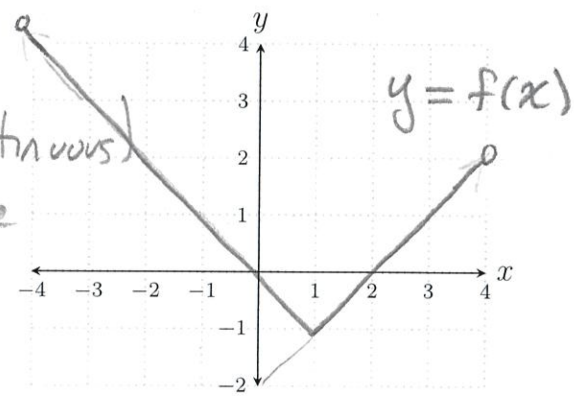
1. $\lim\limits_{x \to 2} \dfrac{7\sin(x-2)}{3x-6} =$ $\lim\limits_{x\to 2} \dfrac{7}{3} \dfrac{\sin(x-2)}{x-2} = \dfrac{7}{3} \lim\limits_{x\to 2} \dfrac{\sin(x-2)}{x-2} = \dfrac{7}{3}\cdot 1 = \boxed{\dfrac{7}{3}}$

2. $\lim\limits_{x \to \pi} \cos\left(\dfrac{x^2-\pi^2}{8(x-\pi)}\right) =$ $\cos\left(\lim\limits_{x\to\pi} \dfrac{x^2-\pi^2}{8(x-\pi)}\right) = \cos\left(\lim\limits_{x\to\pi} \dfrac{\cancel{(x-\pi)}(x+\pi)}{8\cancel{(x-\pi)}}\right)$

$= \cos\left(\lim\limits_{x\to\pi} \dfrac{x+\pi}{8}\right) = \cos\left(\dfrac{\pi+\pi}{8}\right) = \cos\left(\dfrac{\pi}{4}\right) = \boxed{\dfrac{\sqrt{2}}{2}}$

3. A piecewise function f is given below, where the number k is a constant.
(a) Find the value of k for which f is continuous on $(-4, 4)$.
(b) Sketch the graph of this continuous function.

$$f(x) = \begin{cases} -x & \text{if } x < 1 \\ x + k & \text{if } x \geq 1 \end{cases}$$



Since f(x) is a polynomial (i.e. continuous) for $x<1$ or $x>1$, it can only be discontinuous at $x=-1$. For continuity at $x=-1$ we need

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\lim\limits_{x\to 1^-} (-x) = \lim\limits_{x\to 1^+} (x+k)$ $\Longrightarrow$ $-1 = 1+k \Rightarrow \boxed{k=-2}$

$\therefore \boxed{f(x) = \begin{cases} -x & \text{if } x<1 \\ x-2 & \text{if } x\geq 1 \end{cases}}$

4. State the Intermediate Value Theorem.

If f is continuous on $[a,b]$ and y_0 is a number between $f(a)$ and $f(b)$, then there exists a number c in $[a,b]$ for which $f(c) = y_0$.

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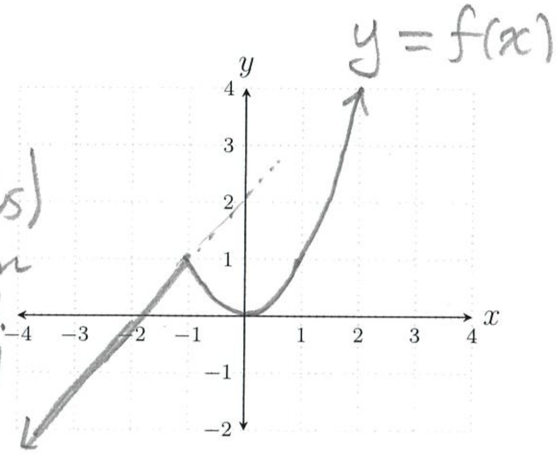
1. $\lim\limits_{x\to 0} \dfrac{\sin(x^2)}{x} = \lim\limits_{x\to 0} \dfrac{\sin(x^2)}{x^2}\cdot x = \lim\limits_{x\to 0}\dfrac{\sin(x^2)}{x^2}\cdot \lim\limits_{x\to 0} x = 1\cdot 0 = \boxed{0}$

2. $\lim\limits_{x\to\sqrt{2}} \tan\left(\dfrac{\pi \log_2(x)}{2}\right) = \tan\left(\lim\limits_{x\to\sqrt{2}} \dfrac{\pi\log_2(x)}{2}\right) = \tan\left(\dfrac{\pi\log_2(\sqrt{2})}{2}\right)$

$= \tan\left(\dfrac{\pi\,\frac{1}{2}}{2}\right) = \tan\left(\dfrac{\pi}{4}\right) = \boxed{1}$

3. A piecewise function f is given below, where the number k is a constant.
   (a) Find the value of k for which f is continuous on $(-4, 4)$.
   (b) Sketch the graph of this continuous function.

$$f(x) = \begin{cases} x + k & \text{if } x < -1 \\ x^2 & \text{if } x \geq -1 \end{cases}$$



Since f is a polynomial (i.e. continuous) for $x < -1$ or $x > -1$, this function could only be discontinuous at $x = -1$. For continuity at $x = -1$ we need

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

$$\lim_{x\to -1}(x+k) = \lim_{x\to -1} x^2 \Rightarrow -1 + k = (-1)^2 \Rightarrow \boxed{k = 2}$$

$$\therefore \boxed{f(x) = \begin{cases} x+2 & \text{if } x < -1 \\ x^2 & \text{if } x \geq 2 \end{cases}}$$

4. State the Intermediate Value Theorem.

If f is continuous on $[a, b]$ and y_0 is a number between $f(a)$ and $f(b)$, then there is a number c in $[a, b]$ for which $f(c) = y_0$.