Name: ______________________________

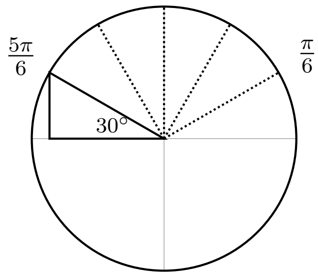
Quiz 3

MATH 200

January 27, 2022

Directions: Find the limits. Show all steps. Simplify your answer.

1. $$\lim_{x\to 5\pi/6} \tan(x) = \tan(5\pi/6) = \frac{\sin(5\pi/6)}{\cos(5\pi/6)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \boxed{-\frac{1}{\sqrt{3}}}$$



2. $$\lim_{x\to 0} \frac{(3x-6)\sin(x)}{x^2-2x} = \lim_{x\to 0} \frac{3(x-2)\sin(x)}{x(x-2)} = \lim_{x\to 0} \frac{3\sin(x)}{x} = 3\lim_{x\to 0} \frac{\sin(x)}{x} = 3\cdot 1 = \boxed{3}$$

3. $$\lim_{x\to 0} \sin^{-1}\left(\frac{\sin(x)}{x}\right) = \sin^{-1}\left(\lim_{x\to 0}\frac{\sin(x)}{x}\right) = \sin^{-1}(1) = \left(\begin{array}{c}\text{angle } \theta \text{ for which}\\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\\ \textbf{and } \sin(\theta) = 1\end{array}\right) = \boxed{\frac{\pi}{2}}$$

4. State the intervals on which the function $f(x) = \dfrac{x}{e^x - 3}$ is continuous.

This is a function x, continuous on its domain (all real numbers) divided by a function $e^x - 3$ continuous on its domain (all real numbers). Therefore the quotient will be continuous on its domain, which is all real numbers except those for which $e^x - 3 = 0$. Solving this,

$$\begin{aligned} e^x - 3 &= 0 \\ e^x &= 3 \\ \ln(e^x) &= \ln(3) \\ x &= \ln(3) \end{aligned}$$

Thus the domain of f is all real numbers except $\ln(3)$.

Answer: $\boxed{f \text{ is continuous on the intervals } (-\infty, \ln(3)) \cup (\ln(3), \infty)}$

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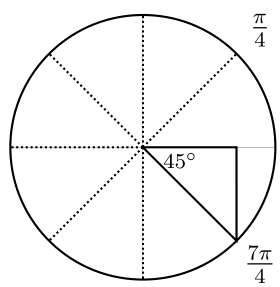
January 27, 2022

Directions: Find the limits. Show all steps. Simplify your answer.

1. $\displaystyle \lim_{x\to\frac{7\pi}{4}} \sec(x) = \sec(7\pi/4) = \frac{1}{\cos(7\pi/4)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$



2. $\displaystyle \lim_{x\to 0} \frac{6\sin(x)}{x^3+7x} = \lim_{x\to 0} \frac{6\sin(x)}{(x^2+7)x} = \lim_{x\to 0} \frac{6}{(x^2+7)}\cdot\frac{\sin(x)}{x} = \lim_{x\to 0} \frac{6}{(x^2+7)}\cdot \lim_{x\to 0}\frac{\sin(x)}{x} = \frac{6}{0^2+7}\cdot 1 = \boxed{\frac{6}{7}}$

3. $\displaystyle \lim_{x\to 2} \tan^{-1}\left(\frac{x^2-3x+2}{x^2-5x+6}\right) = \tan^{-1}\left(\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}\right) = \tan^{-1}\left(\lim_{x\to 2}\frac{(x-1)(x-2)}{(x-3)(x-2)}\right)$

$\displaystyle = \tan^{-1}\left(\lim_{x\to 2}\frac{x-1}{x-3}\right) = \tan^{-1}\left(\frac{2-1}{2-3}\right) = \tan^{-1}\left(\frac{1}{-1}\right) = \tan^{-1}(-1) = \left(\begin{array}{c}\text{angle } \theta \text{ for which}\\ -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}\\ \textbf{and } \tan(\theta) = -1\end{array}\right) = \boxed{-\frac{\pi}{4}}$

4. State the intervals on which the function $f(x) = \dfrac{\sin(x)}{x}$ is continuous. This is a function $\sin(x)$, continuous on its domain (all real numbers) divided by a function x continuous on its domain (all real numbers). Therefore the quotient will be continuous on its domain, which is all real numbers except $x = 0$. Thus the domain of f is all real numbers except 0.

Answer: $\boxed{f \text{ is continuous on the intervals } (-\infty, 0)\cup(0,\infty)}$