Name: Richard

Quiz 4

MATH 200
September 3, 2025

1. Give an example of a function f for which $\lim\limits_{x\to\infty} f(x) = 2$, $\lim\limits_{x\to 1^+} f(x) = \infty$ and $\lim\limits_{x\to 1^-} f(x) = -\infty$.

$$f(x) = \frac{2x}{x-1}$$

2. $$\lim_{x\to\infty} \cos\left(\frac{\pi x^2 + 12x - 15}{x^2 - 1}\right) = \cos\left(\lim_{x\to\infty} \frac{\pi x^2 + 12x - 15}{x^2 - 1}\right)$$

$$= \cos\left(\lim_{x\to\infty} \frac{\pi x^2 + 12x - 15}{x^2 - 1} \cdot \frac{1/x^2}{1/x^2}\right) = \cos\left(\lim_{x\to\infty} \frac{\pi + \frac{12}{x} - \frac{15}{x^2}}{1 - \frac{1}{x^2}}\right)$$

$$= \cos\left(\frac{\pi + 0 - 0}{1 - 0}\right) = \cos(\pi) = \boxed{-1}$$

3. $$\lim_{x\to 2} \frac{1}{x^2 - 4x + 4} = \lim_{x\to 2} \frac{1}{(x-2)(x-2)} = \frac{1}{(x-2)^2} = \infty$$

denominator approaches 0, and is positive.

4. State the asymptotes (both vertical and horizontal, if any) of the function $f(x) = \dfrac{x-4}{x^2 - 3x - 4}$.

$$f(x) = \frac{x-4}{(x-4)(x+1)} = \frac{1}{x+1}$$

Vertical Asymptote(s): $x = -1$ ← because $\lim\limits_{x\to -1^+} f(x) = \infty$

Horizontal Asymptote(s): $y = 0$ ← because $\lim\limits_{x\to\infty} f(x) = 0$

Name: Richard

QUIZ 4

MATH 200
September 2, 2025

1. Give an example of a function f for which $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to 2^+} f(x) = \infty$ and $\lim_{x\to 2^-} f(x) = -\infty$.

$$f(x) = \frac{x}{x-2}$$

2. $$\lim_{x\to 1^+} \frac{1}{x^2-3x+2} = \lim_{x\to 1^+} \frac{1}{(x-1)(x-2)} = \boxed{-\infty}$$

approaches 0, positive

approaches -1 (negative)

denominator approaches 0 and is negative

3. $$\lim_{x\to\infty} \sin\left(\frac{\pi x^2+12x-15}{2x^2-4x+3}\right) = \sin\left(\lim_{x\to\infty} \frac{\pi x^2+12x-15}{2x^2-4x+3}\cdot\frac{1/x^2}{1/x^2}\right)$$

$$= \sin\left(\lim_{x\to\infty} \frac{\pi + \frac{12}{x} - \frac{15}{x^2}}{2-\frac{4}{x}+\frac{3}{x^2}}\right) = \sin\left(\frac{\pi+0-0}{2-0}\right)$$

$$= \sin\left(\frac{\pi}{2}\right) = \boxed{1}$$

4. State the asymptotes (both vertical and horizontal, if any) of the function $f(x) = \dfrac{x-1}{x^2-4x+3}$.

$$f(x) = \frac{x-1}{(x-1)(x-3)} = \frac{1}{x-3}$$

Vertical Asymptote(s): $x = 3$ ← Because $\lim_{x\to 3^+} f(x) = \infty$

Horizontal Asymptote(s): $y = 0$ ← Because $\lim_{x\to\infty} f(x) = 0$