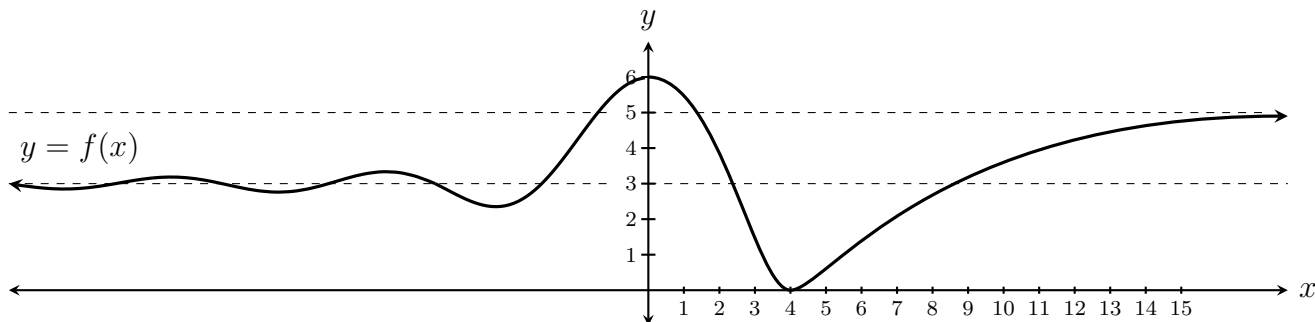


Directions: Find the limits. Show all steps. Simplify your answer.

1. (8 points) Answer the following questions about the function
- $y = f(x)$
- graphed below.



(a) $\lim_{x \rightarrow -\infty} f(x) = \boxed{3}$

(b) $\lim_{x \rightarrow \infty} f(x) = \boxed{5}$

(c) $\lim_{x \rightarrow \infty} \frac{1}{f(x)} = \frac{\lim_{x \rightarrow \infty} 1}{\lim_{x \rightarrow \infty} f(x)} = \boxed{\frac{1}{5}}$

(d) $\lim_{x \rightarrow \infty} f\left(\frac{1}{x}\right) = f\left(\lim_{x \rightarrow \infty} \frac{1}{x}\right) = f(0) = \boxed{6}$

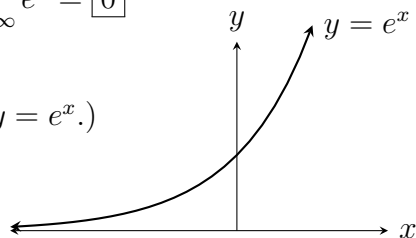
(e) $\lim_{x \rightarrow 4^-} \frac{1}{f(x)} = \boxed{\infty}$ (Bottom approaches 0, and is positive)

(f) $\lim_{x \rightarrow 4^+} \frac{1}{f(x)} = \boxed{\infty}$ (Bottom approaches 0, and is positive)

(g) $\lim_{x \rightarrow 0} \frac{x}{f(x)} = \frac{\lim_{x \rightarrow 0} x}{\lim_{x \rightarrow 0} f(x)} = \frac{0}{6} = \boxed{0}$

(h) $\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = \boxed{\infty}$ (Top approaches 6. Bottom approaches 0, and is positive)

2. (4 points)
- $\lim_{x \rightarrow -\infty} e^x = \boxed{0}$

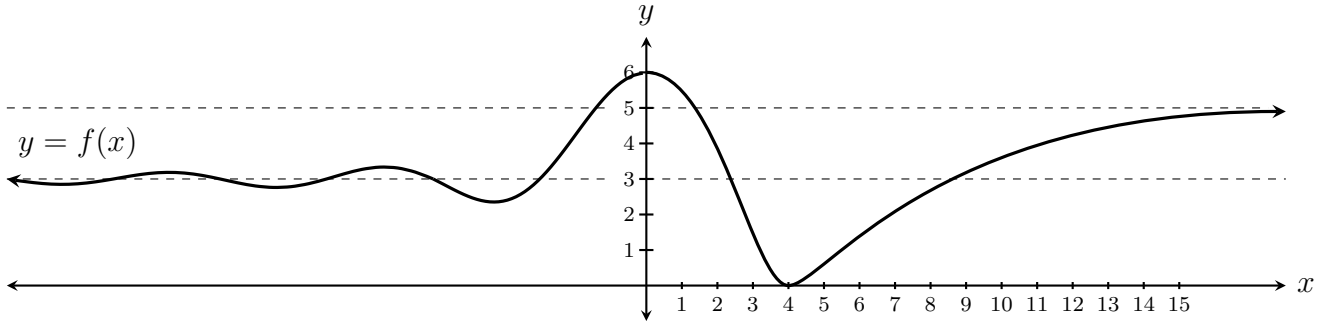
(From graph of $y = e^x$.)

3. (4 points) $\lim_{x \rightarrow 5^+} \frac{x^2 + 2x + 1}{-x^2 + 4x + 5} = \lim_{x \rightarrow 5^+} \frac{(x+1)(x+1)}{(-x+5)(x+1)} = \lim_{x \rightarrow 5^+} \frac{x+1}{-x+5} = \boxed{-\infty}$ (Top approaches 6. Bottom approaches 0, and is negative)

4. (4 points) $\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 1}{-x^2 + 4x + 5} = \lim_{x \rightarrow \infty} \frac{x^2 + 2x + 1}{-x^2 + 4x + 5} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{2}{x} + \frac{1}{x^2}}{-1 + \frac{4}{x} + \frac{5}{x^2}} = \frac{1 + 0 + 0}{-1 - 0} = \boxed{-1}$

Directions: Find the limits. Show all steps. Simplify your answer.

1. (8 points) Answer the following questions about the function $y = f(x)$ graphed below.



(a) $\lim_{x \rightarrow -\infty} f(x) = \boxed{3}$

(b) $\lim_{x \rightarrow \infty} f(x) = \boxed{5}$

(c) $\lim_{x \rightarrow 4^-} \frac{1}{f(x)} = \boxed{\infty}$ (Bottom approaches 0, and is positive)

(d) $\lim_{x \rightarrow 4^+} \frac{1}{f(x)} = \boxed{\infty}$ (Bottom approaches 0, and is positive)

(e) $\lim_{x \rightarrow -\infty} \frac{1}{f(x)} = \frac{\lim_{x \rightarrow -\infty} 1}{\lim_{x \rightarrow -\infty} f(x)} = \boxed{\frac{1}{3}}$

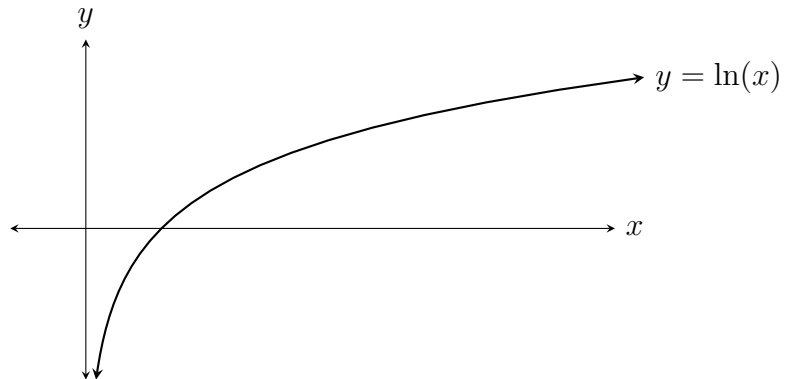
(f) $\lim_{x \rightarrow \infty} f\left(\frac{1}{x} + 4\right) = f\left(\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 4\right)\right) = f(0+4) = f(4) = \boxed{0}$

(g) $\lim_{x \rightarrow 0^-} \frac{f(x)}{x} = \boxed{-\infty}$ (Top approaches 6, Bottom approaches 0, and is negative)

(h) $\lim_{x \rightarrow 0} \frac{x}{f(x)} = \frac{\lim_{x \rightarrow 0} x}{\lim_{x \rightarrow 0} f(x)} = \frac{0}{6} = \boxed{0}$

2. (4 points) $\lim_{x \rightarrow 0^+} \ln(x) = \boxed{-\infty}$

(From graph of $y = \ln(x)$.)



3. (4 points) $\lim_{x \rightarrow \infty} \frac{x^2 + 5x + 6}{x^2 - 9} = \lim_{x \rightarrow \infty} \frac{x^2 + 5x + 6}{x^2 - 9} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{5}{x} + \frac{6}{x^2}}{1 - \frac{9}{x^2}} = \frac{1 + 0 + 0}{1 - 0} = \boxed{1}$

4. (4 points) $\lim_{x \rightarrow 3^+} \frac{x^2 + 5x + 6}{x^2 - 9} = \lim_{x \rightarrow 3^+} \frac{(x + 2)(x + 3)}{(x - 3)(x + 3)} = \lim_{x \rightarrow 3^+} \frac{x + 2}{x - 3} = \boxed{\infty}$ (Top approaches 5, Bottom approaches 0, and is positive)