Name: Richard

QUIZ 5 ♡

MATH 200
September 19, 2022

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x}$.

METHOD A

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x} = \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x} \cdot \frac{zx}{zx}$$

$$= \lim_{z \to x} \frac{x - z}{(z - x)zx} = \lim_{z \to x} \frac{-\cancel{(-x+z)}}{\cancel{(z-x)}zx} = \lim_{z \to x} \frac{-1}{zx}$$

$$= \frac{-1}{x \cdot x} = \boxed{\frac{-1}{x^2}}$$

Answer $\boxed{f'(x) = \frac{-1}{x^2}}$

METHOD B

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{(x+h)x}{(x+h)x}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h(x+h)x} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h)x}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{(x+0)x} = \boxed{\frac{-1}{x^2}} \quad \boxed{f'(x) = \frac{-1}{x^2}}$$

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QUIZ 5 ◊

MATH 200
September 19, 2022

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = 5 - 3x^2$.

METHOD A

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{(5 - 3z^2) - (5 - 3x^2)}{z - x}$$

$$= \lim_{z \to x} \frac{5 - 3z^2 - 5 + 3x^2}{z - x} = \lim_{z \to x} \frac{3x^2 - 3z^2}{z - x}$$

$$= \lim_{z \to x} \frac{3(x^2 - z^2)}{z - x} = \lim_{z \to x} \frac{3\overset{(-1)}{\cancel{(x - z)}}(x + z)}{\cancel{z - x}}$$

$$= \lim_{z \to x} -3(x + z) = -3(x + x) = -3 \cdot 2x = \boxed{-6x}$$

Answer $\boxed{f'(x) = -6x}$

METHOD B

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5 - 3(x+h)^2 - (5 - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{5 - 3(x^2 + 2xh + h^2) - 5 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h} = \lim_{h \to 0} \frac{-6xh - 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-6x - 3h)}{h} = \lim_{h \to 0} (-6x - 3h) = -6x - 3 \cdot 0 = \boxed{-6x}$$

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QUIZ 5 ♣

MATH 200
September 19, 2022

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = 5x^2 - 2$.

METHOD A

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{(5z^2 - 2) - (5x^2 - 2)}{z - x}$$

$$= \lim_{z \to x} \frac{5z^2 - 2 - 5x^2 + 2}{z - x} = \lim_{z \to x} \frac{5z^2 - 5x^2}{z - x}$$

$$= \lim_{z \to x} \frac{5(z^2 - x^2)}{z - x} = \lim_{z \to x} \frac{5\cancel{(z - x)}(z + x)}{\cancel{(z - x)}}$$

$$= \lim_{z \to x} 5(z + x) = 5(x + x) = 5 \cdot 2x = \boxed{10x}$$

$$\boxed{\text{Answer } f'(x) = 10x}$$

Method B

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5(x+h)^2 - 2 - (5x^2 - 2)}{h}$$

$$= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - 2 - 5x^2 + 2}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 2 - 5x^2 + 2}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(10x + 5h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (10x + 5h) = 10x + 5 \cdot 0 = \boxed{10x}$$

$$\boxed{\text{Answer: } f'(x) = 10x}$$

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QUIZ 5 ♠

MATH 200
September 19, 2022

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \sqrt{x}$.

METHOD A

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x} \cdot \frac{\sqrt{z} + \sqrt{x}}{\sqrt{z} + \sqrt{x}}$$

$$= \lim_{z \to x} \frac{\sqrt{z}^2 + \sqrt{z}\sqrt{x} - \sqrt{x}\sqrt{z} + \sqrt{x}^2}{(z - x)(\sqrt{z} + \sqrt{x})}$$

$$= \lim_{z \to x} \frac{\cancel{z - x}}{\cancel{(z - x)}(\sqrt{z} + \sqrt{x})} = \lim_{z \to x} \frac{1}{\sqrt{z} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\boxed{\text{Answer: } f'(x) = \frac{1}{2\sqrt{x}}}$$

METHOD B

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h}^2 + \sqrt{x+h}\sqrt{x} - \sqrt{x}\sqrt{x+h} - \sqrt{x}^2}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \boxed{\frac{1}{2\sqrt{x}}}$$

$$\boxed{\text{Answer: } f'(x) = \frac{1}{2\sqrt{x}}}$$