Name: Richard

QUIZ 5

MATH 200

September 5, 2025

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x}$$

$$= \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x} \cdot \frac{zx}{zx} = \lim_{z \to x} \frac{x - z}{(z - x)zx}$$

$$= \lim_{z \to x} \frac{-\cancel{(z-x)}}{\cancel{(z-x)}zx} = \lim_{z \to x} \frac{-1}{zx} = \frac{-1}{xx} = \frac{-1}{x^2}$$

$$\therefore \boxed{f'(x) = \frac{-1}{x^2}}$$

Alternate method:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} = \lim_{h \to 0} \frac{x - (x+h)}{h\,x(x+h)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\,x(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = \frac{-1}{x^2}$$

$$\therefore \boxed{f'(x) = \frac{-1}{x^2}}$$

Name: Richard

QUIZ 5

MATH 200
September 4, 2025

1. Use a limit definition of the derivative to find the derivative of the function $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{z - x}$$

$$= \lim_{z \to x} \frac{\sqrt{z} - \sqrt{x}}{\sqrt{z}^2 - \sqrt{x}^2} = \lim_{z \to x} \frac{\cancel{\sqrt{z} - \sqrt{x}}}{\cancel{(\sqrt{z} - \sqrt{x})}(\sqrt{z} + \sqrt{x})}$$

$$= \lim_{z \to x} \frac{1}{\sqrt{z} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\therefore \boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

Alternate method:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h}^2 + \sqrt{x+h}\sqrt{x} - \sqrt{x}\sqrt{x+h} - \sqrt{x}^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$