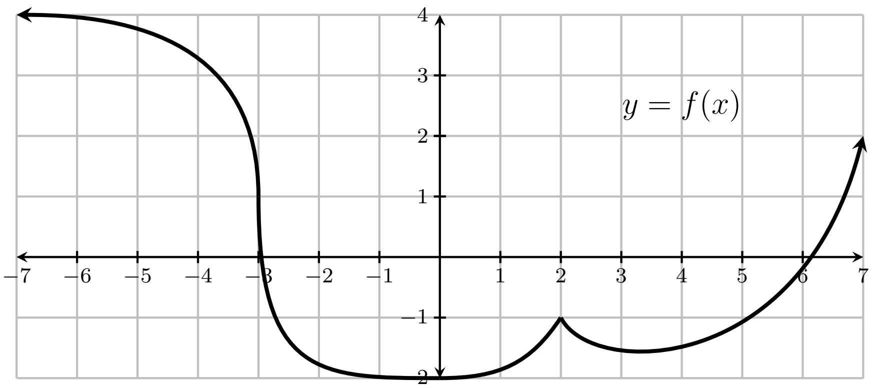
Name: ______________________________ **QUIZ 7** **MATH 200**

February 10, 2022

1. (4 pts.) State the intervals on which the function graphed below is differentiable.



Notice that $f'(-3)$ is not defined as the tangent at $x = -3$ is vertical. Also $f'(2)$ is not defined because there is a cusp at x=2 (so no tangent line there). At all other values of x there is a non-vertical tangent.

Answer:

$f(x)$ is differentiable on $(-7, -3) \cup (-3, 2) \cup (2, 7)$

2. (8 pts.) Consider the functions $f(x) = x^2$ and $g(x) = x^3$. Find all x for which the tangent line to the graph of y=$f(x)$ at $(x, f(x))$ is parallel to the tangent line to the graph of y=$g(x)$ at $(x, g(x))$.

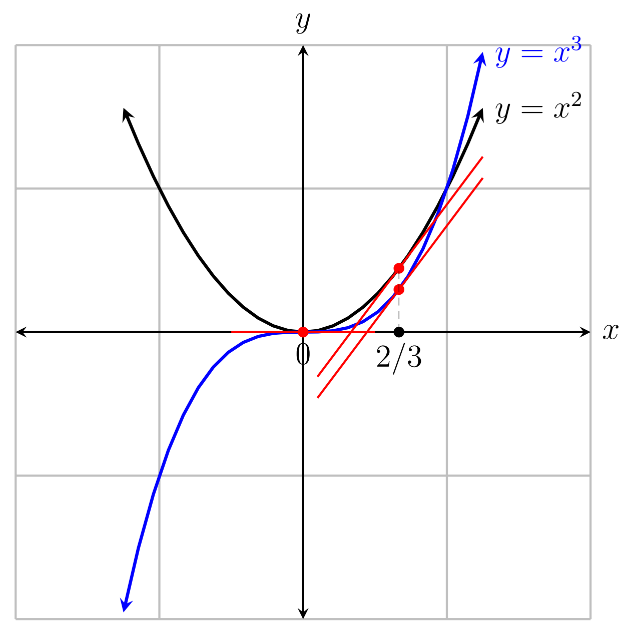
To find x we need to solve the equation $f'(x) = g'(x)$. Now, $f'(x) = 2x$ and $g'(x) = 3x^2$, so we need to solve

$$\begin{aligned} f'(x) &= g'(x) \\ 2x &= 3x^2 \\ 2x - 3x^2 &= 0 \\ x(2-3x) &= 0 \end{aligned}$$

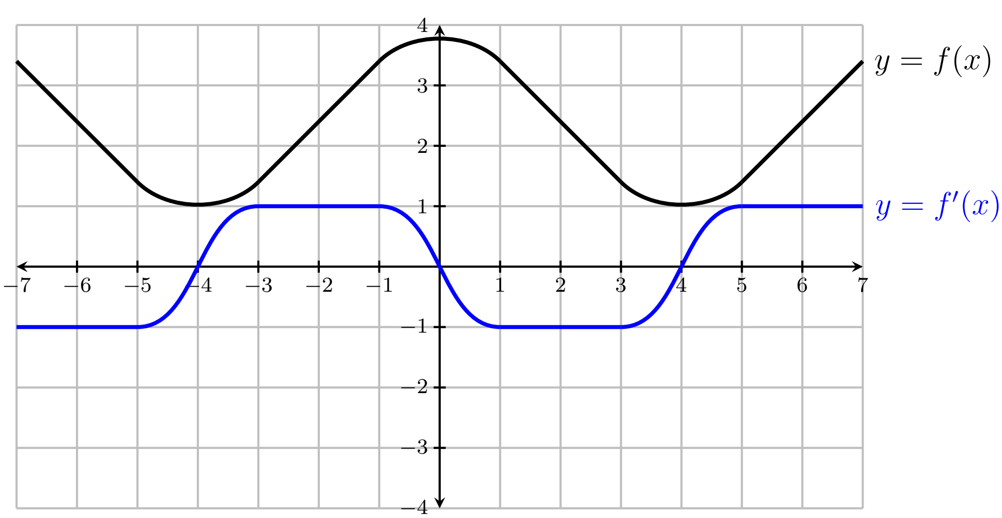
The solutions are $x = 0$ and $x = 2/3$.

So the two graphs have the same slope when x=0 and also when x=2/3.

This is supported by the graphs on the right, which were done with a graphing utility.



3. (8 pts.) The graph of a function $f(x)$ is shown below.
Using the same coordinate axis, sketch the graph of its derivative $f'(x)$



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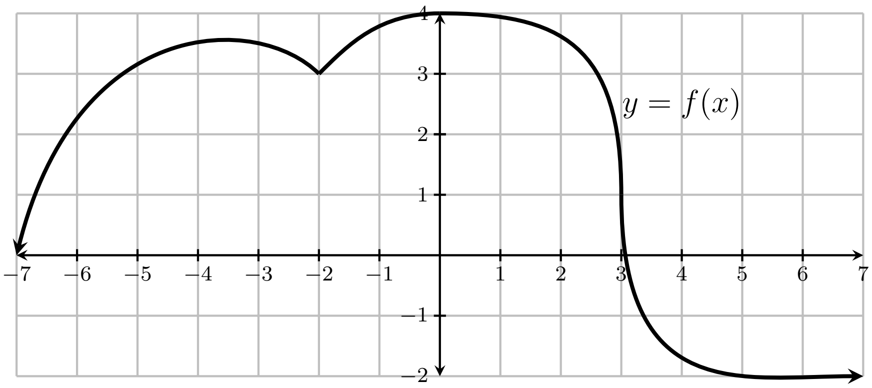


Notice that $f'(3)$ is not defined as the tangent at $x = 3$ is vertical. Also $f'(-2)$ is not defined because there is a cusp at $x = -2$ (so no tangent line there). At all other values of x there is a non-vertical tangent.

Answer:

$f(x)$ is differentiable on $(-7, -2) \cup (-2, 3) \cup (3, 7)$

2. (8 pts.) Consider the functions $f(x) = x^2$ and $g(x) = 4\sqrt{x}$. Find all x for which the tangent line to the graph of $y = f(x)$ at $(x, f(x))$ is parallel to the tangent line to the graph of $y = g(x)$ at $(x, g(x))$.

To find x we need to solve the equation $f'(x) = g'(x)$.

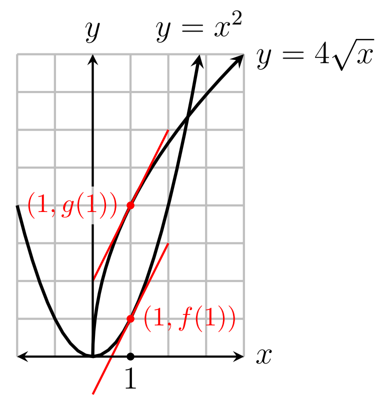
Now, $f'(x) = 2x$ and because $g(x) = 4x^{1/2}$, we get $g'(x) = 4\frac{1}{2}x^{-1/2} = \frac{4}{2x^{1/2}} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$.

Now let's solve $f'(x) = g'(x)$.

$$\begin{aligned}
2x &= \frac{2}{\sqrt{x}} \\
x &= \frac{1}{\sqrt{x}} \\
x\sqrt{x} &= 1 \\
x^1 x^{1/2} &= 1 \\
x^{1+1/2} &= 1 \\
x^{3/2} &= 1 \\
\left(x^{3/2}\right)^{2/3} &= 1^{2/3} \\
x &= 1
\end{aligned}$$

So the two graphs have the same slope when $x = 1$.

This is supported by the graphs below, which were done with a graphing utility. (Not that that was available to you on the quiz!) Note that the slopes do appear to be equal at $x = 1$.



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