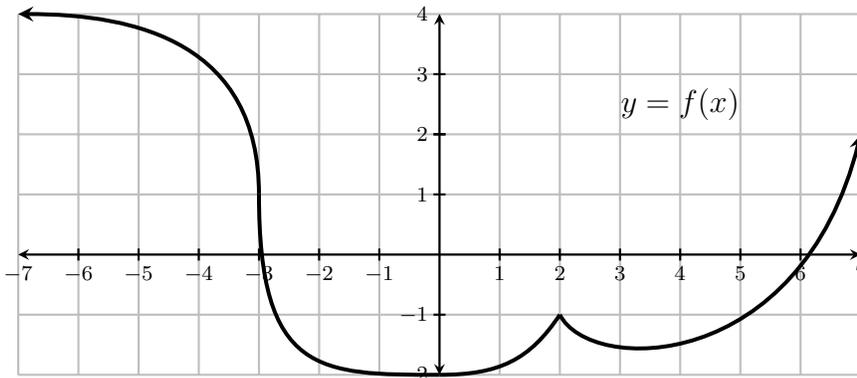


Name: \_\_\_\_\_

1. (4 pts.) State the intervals on which the function graphed below is differentiable.



Notice that  $f'(-3)$  is not defined as the tangent at  $x = -3$  is vertical. Also  $f'(2)$  is not defined because there is a cusp at  $x = 2$  (so no tangent line there). At all other values of  $x$  there is a non-vertical tangent.

**Answer:**

$f(x)$  is differentiable on  $(-7, -3) \cup (-3, 2) \cup (2, 7)$

2. (8 pts.) Consider the functions  $f(x) = x^2$  and  $g(x) = x^3$ . Find all  $x$  for which the tangent line to the graph of  $y=f(x)$  at  $(x, f(x))$  is parallel to the tangent line to the graph of  $y=g(x)$  at  $(x, g(x))$ .

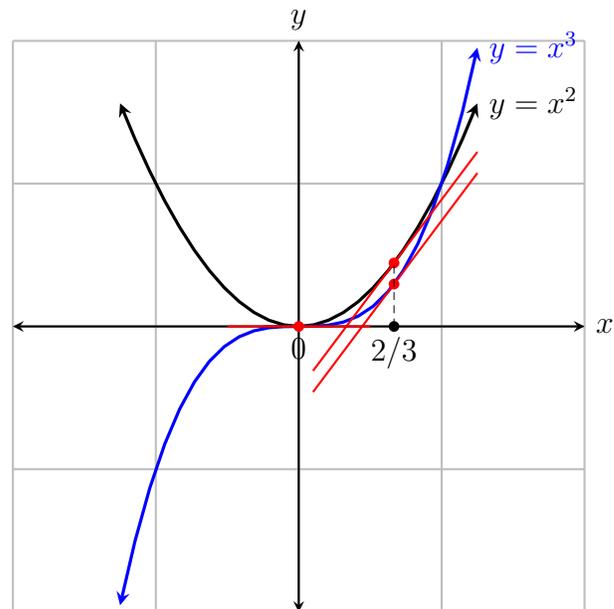
To find  $x$  we need to solve the equation  $f'(x) = g'(x)$ . Now,  $f'(x) = 2x$  and  $g'(x) = 3x^2$ , so we need to solve

$$\begin{aligned} f'(x) &= g'(x) \\ 2x &= 3x^2 \\ 2x - 3x^2 &= 0 \\ x(2 - 3x) &= 0 \end{aligned}$$

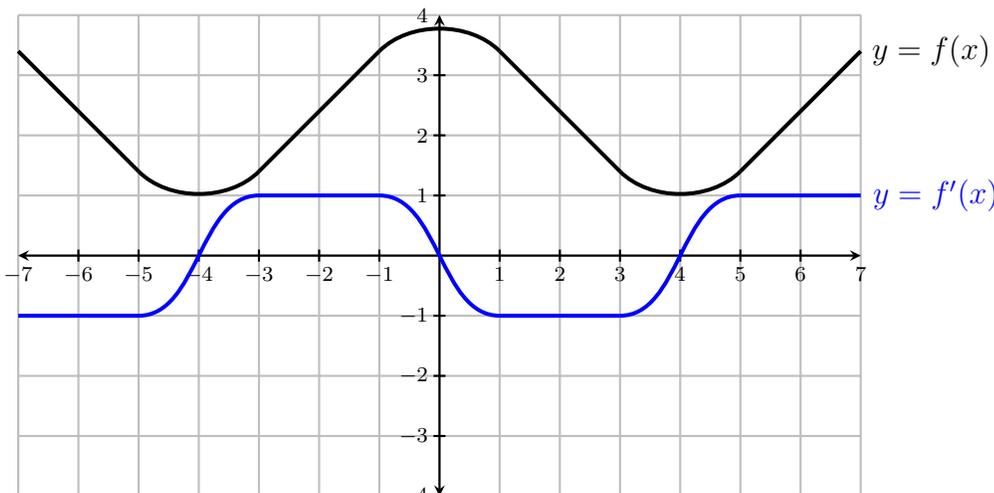
The solutions are  $x = 0$  and  $x = 2/3$ .

So the two graphs have the same slope when  $x=0$  and also when  $x=2/3$ .

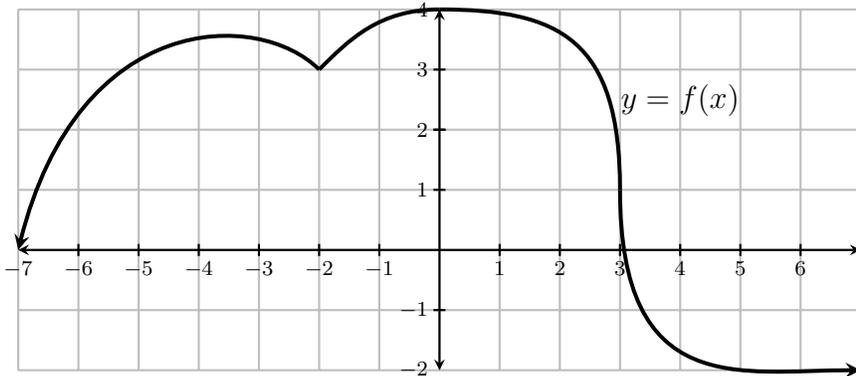
This is supported by the graphs on the right, which were done with a graphing utility.



3. (8 pts.) The graph of a function  $f(x)$  is shown below. Using the same coordinate axis, sketch the graph of its derivative  $f'(x)$



1. (4 pts.) State the intervals on which the function graphed below is differentiable.



Notice that  $f'(3)$  is not defined as the tangent at  $x=3$  is vertical. Also  $f'(-2)$  is not defined because there is a cusp at  $x=-2$  (so no tangent line there). At all other values of  $x$  there is a non-vertical tangent.

**Answer:**

$f(x)$  is differentiable on  $(-7, -2) \cup (-2, 3) \cup (3, 7)$

2. (8 pts.) Consider the functions  $f(x) = x^2$  and  $g(x) = 4\sqrt{x}$ . Find all  $x$  for which the tangent line to the graph of  $y=f(x)$  at  $(x, f(x))$  is parallel to the tangent line to the graph of  $y=g(x)$  at  $(x, g(x))$ .

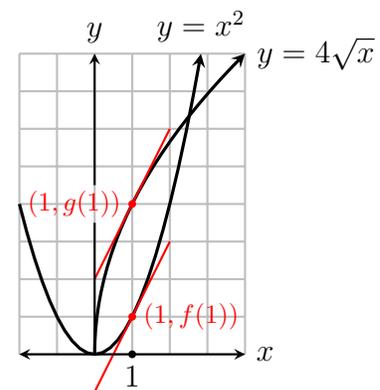
To find  $x$  we need to solve the equation  $f'(x) = g'(x)$ .

Now,  $f'(x) = 2x$  and because  $g(x) = 4x^{1/2}$ , we get  $g'(x) = 4 \cdot \frac{1}{2}x^{-1/2} = \frac{4}{2x^{1/2}} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$ .

Now let's solve  $f'(x) = g'(x)$ .

$$\begin{aligned} 2x &= \frac{2}{\sqrt{x}} \\ x &= \frac{1}{\sqrt{x}} \\ x\sqrt{x} &= 1 \\ x^1 x^{1/2} &= 1 \\ x^{1+1/2} &= 1 \\ x^{3/2} &= 1 \\ (x^{3/2})^{2/3} &= 1^{2/3} \\ x &= 1 \end{aligned}$$

This is supported by the graphs below, which were done with a graphing utility. (Not that that was available to you on the quiz!) Note that the slopes do appear to be equal at  $x=1$ .



So the two graphs have the same slope when  $x=1$ .

3. (8 pts.) The graph of a function  $f(x)$  is shown below. Using the same coordinate axis, sketch the graph of its derivative  $f'(x)$ .

