Name: Richard

QUIZ 9 ♡

MATH 200
September 29, 2021

1. In this problem $y = x\sin(x)$.

(a) $\frac{dy}{dx} = 1\cdot\sin(x) + x\cos(x) = \boxed{\sin(x) + x\cos(x)}$ (product rule)

(b) $\frac{d^2y}{dx^2} = \cos(x) + 1\cdot\cos(x) + x(-\sin(x)) = \boxed{2\cos(x) - x\sin(x)}$

(c) $\frac{d^3y}{dx^3} = -2\sin(x) - (1\cdot\sin(x) + x\cos(x)) = \boxed{-3\sin(x) - x\cos(x)}$

2. Find the derivative of $y = \tan(3x^2 + x)$.

$$\frac{dy}{dx} = \boxed{\sec^2(3x^2+x)(6x+1)}$$

(chain rule)

3. Find the derivative of $y = \cos\left(\frac{1}{x}\right)$. $= \cos(x^{-1})$

$$\frac{dy}{dx} = -\sin\left(\frac{1}{x}\right)\left(-1\cdot x^{-1-1}\right) = -\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$

(chain rule)

$$= \boxed{\frac{\sin\left(\frac{1}{x}\right)}{x^2}}$$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given in the table below. If $h(x) = f(g(x))$, find $h'(3)$.

x	0	1	2	3	4	5
$f(x)$	-4	-2	0	1	1	0
$f'(x)$	2	1	1	3	5	-1
$g(x)$	10	9	7	4	0	-4
$g'(x)$	0	-0.5	-1	-3	-4	-4

$h'(x) = f'(g(x))\,g'(x)$

$h'(3) = f'(g(3))\,g'(3)$

$= f'(4)(-3)$

$= 5\cdot(-3) = \boxed{-15}$

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QUIZ 9 ♣

MATH 200
September 29, 2021

1. In this problem $y = xe^x$.

(a) $\frac{dy}{dx} = 1 \cdot e^x + x e^x = \boxed{e^x + xe^x}$ (product rule)

(b) $\frac{d^2y}{dx^2} = e^x + 1 \cdot e^x + x e^x = \boxed{2e^x + xe^x}$

(c) $\frac{d^3y}{dx^3} = 2e^x + 1 \cdot e^x + x e^x = \boxed{3e^x + xe^x}$

2. Find the derivative of $y = \sin\left(\sqrt{x}\right)$. $= \sin\left(x^{\frac{1}{2}}\right)$

$$\frac{dy}{dx} = \cos\left(x^{\frac{1}{2}}\right) \frac{1}{2} x^{\frac{1}{2}-1} = \boxed{\cos\left(\sqrt{x}\right)\frac{1}{2\sqrt{x}}}$$

(chain rule)

3. Find the derivative of $y = \tan\left(3x^3 + x\right)$.

$$\frac{dy}{dx} = \boxed{\sec^2\left(3x^3 + x\right)\left(9x^2 + 1\right)}$$

(chain Rule)

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given in the table below. If $h(x) = f\left(g(x)\right)$, find $h'(4)$.

x	0	1	2	3	4	5
$f(x)$	−4	−2	0	1	1	0
$f'(x)$	2	1	1	3	0.5	−1
$g(x)$	10	9	7	4	0	−4
$g'(x)$	0	−0.5	−1	−3	−4	−4

$h'(x) = f'(g(x))g'(x)$

$h'(4) = f'(g(4))g'(4)$

$= f'(0)(-4) = 2 \cdot (-4)$

$= \boxed{-8}$

Name: Richard

QUIZ 9 ◊

MATH 200
September 29, 2021

1. In this problem $y = \frac{2}{x^2}$. $= 2x^{-2}$

(a) $\frac{dy}{dx} = -4x^{-3} = \boxed{\frac{-4}{x^3}}$

(b) $\frac{d^2y}{dx^2} = 12x^{-4} = \boxed{\frac{12}{x^4}}$

(c) $\frac{d^3y}{dx^3} = -48x^{-5} = \boxed{\frac{-48}{x^5}}$

2. Find the derivative of $y = \cos(xe^x)$.

$y' = -\sin(xe^x)\, D_x[xe^x]$ ← (chain rule)

$= -\sin(xe^x)(1 \cdot e^x + xe^x)$ ← (product rule)

$= \boxed{-\sin(xe^x)(e^x + xe^x)}$

3. Find the derivative of $y = \cot(3x^2 + x)$.

$y' = \boxed{-\csc^2(3x^2+x)(6x+1)}$ ← (chain rule)

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given in the table below.
If $h(x) = f(g(x))$, find $h'(0)$.

x	0	1	2	3	4	5
$f(x)$	−4	−2	0	1	1	0
$f'(x)$	2	1	1	3	0.5	−1
$g(x)$	5	9	7	4	0	−4
$g'(x)$	3	−0.5	−1	−3	−4	−4

$h'(x) = f'(g(x))g'(x)$ (chain rule)

$h'(0) = f'(g(0))g'(0)$

$= f'(5) \cdot 3$

$= (-1) \cdot 3 = \boxed{-3}$

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QUIZ 9 ♠

MATH 200
September 29, 2021

1. In this problem $y = x^2 + \frac{1}{x}$. $= x^2 + x^{-1}$

(a) $\frac{dy}{dx} = 2x - x^{-2} = \boxed{2x - \frac{1}{x^2}}$

(b) $\frac{d^2y}{dx^2} = 2 - (-2)x^{-3} = \boxed{2 + \frac{2}{x^3}}$

(c) $\frac{d^3y}{dx^3} = 0 - 6x^{-4} = \boxed{\frac{-6}{x^4}}$

2. Find the derivative of $y = \sin(x^2e^x)$.

Chain rule: $y' = \cos(x^2e^x)\, D_x[x^2e^x]$

$= \boxed{\cos(x^2e^x)(2xe^x + x^2e^x)}$

3. Find the derivative of $y = \tan\left(\frac{1}{x^2}\right)$. $= \tan(x^{-2})$

Chain rule: $y' = \sec^2\left(\frac{1}{x^2}\right)(-2)x^{-2-1}$

$= \boxed{-\frac{2\sec^2\left(\frac{1}{x}\right)}{x^3}}$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given in the table below. If $h(x) = f(g(x))$, find $h'(1)$.

x	0	1	2	3	4	5
$f(x)$	-4	-2	0	1	1	0
$f'(x)$	2	1	1	3	6	-1
$g(x)$	10	4	7	4	0	-4
$g'(x)$	0	-0.5	-1	-3	-4	-4

$h'(x) = f'(g(x)) \cdot g'(x)$ (chain rule)

$h'(1) = f'(g(1))\, g'(1)$

$= f'(4)(-0.5)$

$= 6 \cdot (-0.5) = \boxed{-3}$