Name: Richard

QUIZ 9 ♡

MATH 200
October 3, 2022

1. In this problem $y = \cos(3x+1)$.

   (a) $\frac{dy}{dx} = -\sin(3x+1)\, D_x[3x+1] = -\sin(3x+1)\cdot 3 = \boxed{-3\sin(3x+1)}$

   (b) $\frac{d^2y}{dx^2} = -3\left(\cos(3x+1)\, D_x[3x+1]\right) = \boxed{-9\cos(3x+1)}$

2. Find the derivative of $y = \tan(x^3 - 5x^2 + 3)$.

$y' = \boxed{\sec^2(x^3 - 5x^2 + 3)(3x^2 - 10x)}$

3. Find the derivative of $y = \sin(2e^x)$.

$y' = \cos(2e^x)\, D_x[2e^x] = \cos(2e^x)\cdot 2e^x$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given below. Let $h(x) = f(g(x))$.

x	1	2	3	4	5	6
$f(x)$	−3	−2	1	5	6	3
$f'(x)$	4	3	2	1	0	−2
$g(x)$	1	1	−2	3	−4	5
$g'(x)$	2	−3	5	−8	10	−15

   (a) Find $h'(4)$.

$h'(x) = f'(g(x))\cdot g'(x)$

$h'(4) = f'(3)\cdot(-8)$

$= 2(-8) = \boxed{-16}$

   (b) Find $h(4)$.

$h(4) = f(g(4)) = f(3) = \boxed{1}$

   (c) Find the **equation** of the tangent line to the graph of $y = h(x)$ at $(4, h(4))$.

Point $(4, h(4)) = (4, 1)$

Slope: $h'(4) = -16$

Point slope formula: $y - y_0 = m(x - x_0)$

$y - 1 = -16(x-4) \longrightarrow \boxed{y = -16x + 65}$

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QUIZ 9 ♣

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1. In this problem $y = \sin(x^2)$.

(a) $\frac{dy}{dx} =$ $\cos(x^2)\,2x \quad = 2x\cos(x^2)$

(b) $\frac{d^2y}{dx^2} =$ $2\cos(x^2) + 2x\left(-\sin(x^2)\,2x\right)$

$= \boxed{2\cos(x^2) - 4x^2\sin(x^2)}$

2. Find the derivative of $y = \cos(\sqrt{x})$.

$D_x\left[\cos(\sqrt{x})\right] = D_x\left[\cos(x^{1/2})\right] =$

$-\sin(x^{1/2})\,\frac{1}{2}x^{\frac{1}{2}-1} = \frac{-\sin(\sqrt{x})}{2x^{1/2}} = \boxed{\frac{-\sin(\sqrt{x})}{2\sqrt{x}}}$

3. Find the derivative of $y = \tan(x^3 - 5x^2 + 3)$.

$D_x\left[\tan(x^3 - 5x^2 + 3)\right] = \boxed{\sec^2(x^3 - 5x^2 + 3)(3x^2 - 10x)}$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given below. Let $h(x) = f(g(x))$.

x	1	2	3	4	5	6
$f(x)$	-3	-2	1	5	6	3
$f'(x)$	4	3	2	1	0	-2
$g(x)$	1	1	-2	3	-4	5
$g'(x)$	2	-3	5	-8	10	-15

(a) Find $h'(2)$.

$h'(x) = D_x\left[f(g(x))\right] = f'(g(x))\,g'(x)$

$h'(2) = f'(g(2))\,g'(2) = f'(1)\cdot(-3)$

$= 4(-3) = \boxed{-12}$

(b) Find $h(2)$.

$h(2) = f(g(2)) = f(1) = \boxed{-3}$

(c) Find the **equation** of the tangent line to the graph of $y = h(x)$ at $(2, h(2))$.

Point: $(2, h(2)) = (2, -3)$

Slope: $h'(2) = -12$

Point-slope formula $y - y_0 = m(x - x_0)$

$y - (-3) = -12(x - 2) \longrightarrow \boxed{y = -12x + 21}$

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QUIZ 9 ◇

MATH 200
October 3, 2022

1. In this problem $y = \cos(2x+1)$.

(a) $\frac{dy}{dx} = -\sin(2x+1)(2+0) = \boxed{-2\sin(2x+1)}$

(b) $\frac{d^2y}{dx^2} = -2\left(\cos(2x+1)(2+0)\right) = \boxed{-4\cos(2x+1)}$

2. Find the derivative of $y = \sin\left(x^5 - x + 5\right)$.

$\frac{dy}{dx} = \boxed{\cos(x^5 - x + 5)(5x^4 - 1)}$

3. Find the derivative of $y = \tan\left(2e^x + x^2\right)$.

$\frac{dy}{dx} = \boxed{\sec^2(2e^x + x^2)(2e^x + 2x)}$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given below. Let $h(x) = f(g(x))$.

x	1	2	3	4	5	6
$f(x)$	−3	−2	1	5	6	3
$f'(x)$	4	3	2	1	−1	−2
$g(x)$	1	1	−2	3	−4	5
$g'(x)$	2	−3	5	−8	10	−15

(a) Find $h'(6)$.

$h'(x) = D_x\left[f(g(x))\right] = f'(g(x))g'(x)$

$h'(6) = f'(g(6))g'(6) = f'(5)\cdot(-15)$

$= (-1)(-15) = \boxed{15}$

(b) Find $h(6)$.

$h(6) = f(g(6)) = f(5) = \boxed{6}$

(c) Find the **equation** of the tangent line to the graph of $y = h(x)$ at $(6, h(6))$.

Point: $(x_0, y_0) = (6, h(6)) = (6, 6)$

Slope: $m = h'(6) = 15$

Point-Slope Formula: $y - y_0 = m(x - x_0)$

$\Rightarrow y - 6 = 15(x-6) \Rightarrow y - 6 = 15x - 90 \quad \boxed{y = 15x - 84}$

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QUIZ 9 ♠

MATH 200
October 3, 2022

1. In this problem $y = \cos(x^2)$.

(a) $\frac{dy}{dx} = -\sin(x^2)\,2x = \boxed{-2x\sin(x^2)}$ (Chain Rule)

(b) $\frac{d^2y}{dx^2} = -2\sin(x^2) - 2x\cos(x^2)\,2x$ (Product Rule and Chain Rule)

$= \boxed{-2\sin(x^2) - 4x^2\cos(x^2)}$

2. Find the derivative of $y = \tan(\sqrt{x})$. $= \tan(x^{1/2})$

$D_x\left[\tan(x^{\frac{1}{2}})\right] = \sec^2(x^{1/2})\,\frac{1}{2}x^{-1/2} = \sec^2(\sqrt{x})\,\frac{1}{2x^{1/2}}$

$= \boxed{\frac{\sec^2(\sqrt{x})}{2\sqrt{x}}}$

3. Find the derivative of $y = \sin(x^3 - 5x^2 + 3)$.

$\frac{dy}{dx} = \boxed{\cos(x^3 - 5x^2 + 3)(3x^2 - 10x)}$

4. Information about functions $f(x)$, $g(x)$ and their derivatives is given below. Let $h(x) = f(g(x))$.

x	1	2	3	4	5	6
$f(x)$	-3	-2	1	5	6	3
$f'(x)$	4	3	2	1	0	-2
$g(x)$	1	1	-2	3	-4	5
$g'(x)$	2	-3	5	-8	10	-15

(a) Find $h'(1)$.

$h'(x) = D_x\left[f(g(x))\right]$

$= f'(g(x))\,g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(1)(\cdot 2)$

$= 4 \cdot 2 = \boxed{8}$

(b) Find $h(1)$.

$h(1) = f(g(1)) = f(1) = \boxed{-3}$

(c) Find the **equation** of the tangent line to the graph of $y = h(x)$ at $(1, h(1))$.

Point: $(1, h(1)) = (1, -3) = (x_0, y_0)$

Slope: $m = h'(1) = 8$

Point-Slope formula: $y - y_0 = m(x - x_0)$

$y - (-3) = 8(x - 1) \longrightarrow y = 8x - 11$