Name: Richard

QUIZ 9

MATH 200
September 21, 2023

1. (6 pts.) Find the derivative of $f(x) = \cos\left(3 + 2x + 4x^3\right)$.

Chain Rule: $f'(x) = -\sin\left(3+2x+4x^3\right) D_x\left[3+2x+4x^3\right]$

$= -\sin\left(3+2x+4x^3\right)\left(0+2+12x^2\right)$

$= \boxed{-\left(2+12x^2\right)\sin\left(3+2x+4x^3\right)}$

2. (7 pts.) If $y = x^2 \tan\left(x^2\right)$, find $\dfrac{dy}{dx}$.

Product Rule: $D_x\left[x^2 \tan(x^2)\right]$

$= D_x\left[x^2\right]\tan(x^2) + x^2 D_x\left[\tan(x^2)\right]$

chain rule

$= 2x\tan(x^2) + x^2\sec^2(x^2)2x$

$= \boxed{2x\tan(x^2) + 2x^3\sec^2(x^2)}$

3. (7 pts.) Suppose $f(x)$ is a function for which $f\left(\frac{\pi}{3}\right) = 4$, $f'\left(\frac{\pi}{3}\right) = -2$ and $f''\left(\frac{\pi}{3}\right) = 6$.
Let $g(x) = f(x)\sin(x)$. Find the exact value of $g''\left(\frac{\pi}{3}\right)$.

$g'(x) = f'(x)\sin(x) + f(x)\cos(x)$ (by product rule)

$g''(x) = f''(x)\sin(x) + f'(x)\cos(x) + f'(x)\cos(x) - f(x)\sin(x)$

$$\boxed{g''(x) = f''(x)\sin(x) + 2f'(x)\cos(x) - f(x)\sin(x)}$$

$g''\left(\frac{\pi}{3}\right) = f''\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right) + 2f'\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$

$= 6\cdot\frac{\sqrt{3}}{2} + 2(-2)\frac{1}{2} - 4\frac{\sqrt{3}}{2} = 3\sqrt{3} - 2 - 2\sqrt{3} = \boxed{\sqrt{3}-2}$

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1. (6 pts.) Find the derivative of $f(x) = \tan\left(3 + 2x + 4x^3\right)$.

Chain rule $f'(x) = \sec^2(3+2x+4x^3)\, D_x[3+2x+4x^3]$

$= \sec^2(3+2x+4x^3)(0+2+12x^2)$

$= \boxed{(2+12x^2)\sec^2(3+2x+4x^3)}$

(need chain rule here)

2. (7 pts.) Suppose $y = \dfrac{\sin(x^2)}{1+e^x}$. Find $\dfrac{dy}{dx}$.

Quotient Rule $\dfrac{dy}{dx} = \dfrac{D_x[\sin(x^2)](1+e^x) - \sin(x^2)D_x[1+e^x]}{(1+e^x)^2}$

$= \dfrac{\cos(x^2)2x(1+e^x) - \sin(x^2)(0+e^x)}{(1+e^x)^2}$

$= \boxed{\dfrac{2x(1+e^x)\cos(x^2) - e^x\sin(x^2)}{(1+e^x)^2}}$

3. (7 pts.) Suppose $f(x)$ is a function for which $f\left(\frac{\pi}{3}\right) = 4$, $f'\left(\frac{\pi}{3}\right) = -2$ and $f''\left(\frac{\pi}{3}\right) = 6$.
Let $g(x) = f(x)\sin(x)$. Find the exact value of $g''\left(\frac{\pi}{3}\right)$.

$g'(x) = f'(x)\sin(x) + f(x)\cos(x)$

$g''(x) = f''(x)\sin(x) + f'(x)\cos(x) + f'(x)\cos(x) - f(x)\sin(x)$

$\boxed{g''(x) = f''(x)\sin(x) + 2f'(x)\cos(x) - f(x)\sin(x)}$

$g''(\pi/3) = f''\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right) + 2f'\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$

$= 6\cdot\frac{\sqrt{3}}{2} + 2(-2)\cdot\frac{1}{2} - 4\frac{\sqrt{3}}{2}$

$= 3\sqrt{3} - 2 - 2\sqrt{3} = \boxed{\sqrt{3} - 2}$