Name: ______________________________

QUIZ 9

MATH 200
February 17, 2022

1. (5 points) In this problem $y = x^2 + e^x$.

   (a) $\dfrac{dy}{dx} = \boxed{2x + e^x}$

   (b) $\dfrac{d^2y}{dx^2} = \boxed{2 + e^x}$

   (c) $\dfrac{d^3y}{dx^3} = \boxed{e^x}$

2. (10 points) This problem concerns the function $f(x) = \sin\left(x^2\right)$.

   (a) Find $f'(x)$. By the chain rule, $\boxed{f'(x) = \cos\left(x^2\right) 2x.}$

   (b) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $\left(\sqrt{\pi}, f(\sqrt{\pi})\right)$.

   A point on the tangent line is $\left(\sqrt{\pi}, f(\sqrt{\pi})\right) = \left(\sqrt{\pi}, \sin(\sqrt{\pi}^2)\right) = \left(\sqrt{\pi}, \sin(\pi)\right) = \boxed{(\sqrt{\pi}, 0)}$

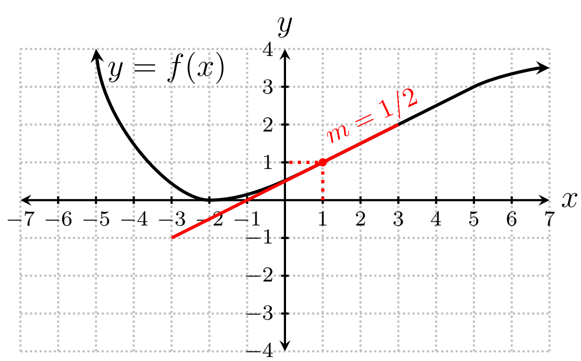
   The slope of the line is $f'(\sqrt{\pi}) = \cos\left(\sqrt{\pi}^2\right) 2\sqrt{\pi} = \cos(\pi)\, 2\sqrt{\pi} = -1 \cdot 2\sqrt{\pi} = \boxed{-2\sqrt{\pi}}$

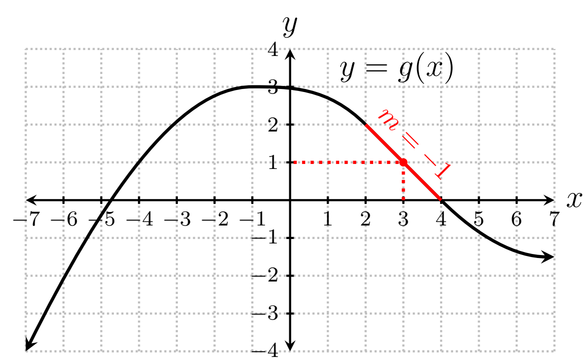
   By the point-slope formula, the equation for the tangent line is

$$
\begin{aligned}
y - y_0 &= m(x - x_0) \\
y - 0 &= -2\sqrt{\pi}\left(x - \sqrt{\pi}\right) \\
y &= -2\sqrt{\pi}\, x + 2\sqrt{\pi}\sqrt{\pi} \\
y &= -2\sqrt{\pi}\, x + 2\pi
\end{aligned}
$$

   Answer: $\boxed{\text{The equation of the tangent line is } y = -2\sqrt{\pi}\, x + 2\pi.}$

3. (5 points) Two functions $f(x)$ and $g(x)$ are graphed below. Suppose $h(x) = f\left(g(x)\right)$. Find $h'(3)$. Please show your work carefully.





By the chain rule, $h'(x) = f'(g(x))g'(x)$.

Thus $h'(3) = f'(g(3))g'(3) = f'(1) \cdot (-1) = \frac{1}{2} \cdot (-1) = \boxed{-\frac{1}{2}}$

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Quiz 9

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February 17, 2022

1. (5 points) In this problem $y = 2x + \cos(x)$

   (a) $\dfrac{dy}{dx} = \boxed{2 - \sin(x)}$

   (b) $\dfrac{d^2y}{dx^2} = \boxed{-\cos(x)}$

   (c) $\dfrac{d^3y}{dx^3} = \boxed{\sin(x)}$

2. (10 points) This problem concerns the function $f(x) = \sin(\pi e^x)$.

   (a) Find $f'(x)$. By the chain rule $f'(x) = \cos(\pi e^x)\, D_x\big[\pi e^x\big] = \boxed{\cos(\pi e^x)\,\pi e^x}$

   (b) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $\big(0, f(0)\big)$.

   A point on the tangent line is $\big(0, f(0)\big) = \big(0, \sin(\pi e^0)\big) = \big(0, \sin(\pi{\cdot}1)\big) = \boxed{(0,0)}$

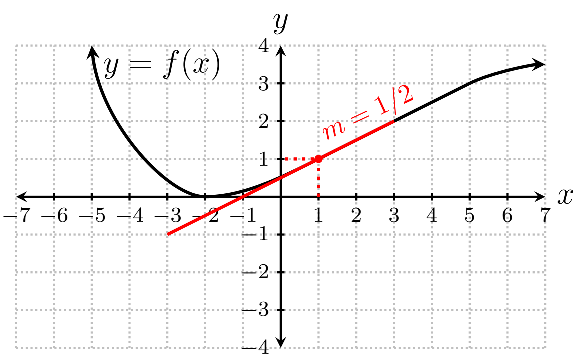
   The slope of the line is $f'(0) = \cos(\pi e^0)\,\pi e^0 = \cos(\pi{\cdot}1)\cdot\pi{\cdot}1 = \cos(\pi){\cdot}\pi = \boxed{-\pi}$

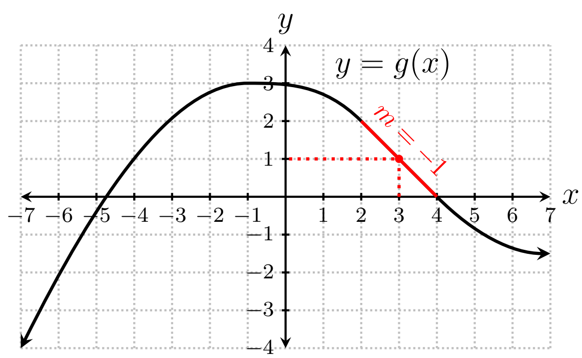
   By the point-slope formula, the equation for the tangent line is

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 0 &= -\pi\big(x - 0\big) \\ y &= -\pi\, x \end{aligned}$$

   Answer: $\boxed{\text{The equation of the tangent line is } y = -\pi\, x.}$

3. (5 points) Two functions $f(x)$ and $g(x)$ are graphed below. Suppose $h(x) = f\big(g(x)\big)$. Find $h'(3)$. Please show your work carefully.





By the chain rule, $h'(x) = f'(g(x))g'(x)$.

Thus $h'(3) = f'(g(3))g'(3) = f'(1)\cdot(-1) = \frac{1}{2}\cdot(-1) = \boxed{-\frac{1}{2}}$