Name: ____________________________

MATH 200 – FINAL EXAM
R. Hammack December 11, 2025

This is a closed-notes, closed book exam. No calculators, no computers, etc. Put phones away.
Answer the questions in the space provided, sowing work. Put your final answer in a box when appropriate.

1. Find the limits using any appropriate method. Give an answer of ∞ or $-\infty$ if necessary.

   (a) $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{x-1}$

   (b) $\displaystyle\lim_{x\to 2} \frac{3}{(x-2)^2}$

2. Find the derivatives of the functions below. You do NOT need to simplify your answer.

   (a) $y = \dfrac{6-x^3}{x+5}$

   (b) $y = e^x - x^e + e^e$

   (c) $y = \sin^{-1}(x) \cdot \sec(x)$

   (d) $y = \big(\ln(x^2+4)\big)^3$

3. Given the equation $6x^2 + \ln(y) = 5 + 3y^2$, find y'

4. Information about two functions f and g is given in the table below.

x	0	3	5	8
$f(x)$	9	6	5	4
$f'(x)$	2	-7	4	7
$g(x)$	2	3	0	0
$g'(x)$	6	5	3	9

(a) Suppose $h(x) = \dfrac{f(x)}{g(x)}$. Find $h'(0)$.

(b) Let k be a function for which $k'(x) = f\big(g(x)\big)$. Is the graph of $k(x)$ concave up or down at $x = 5$?

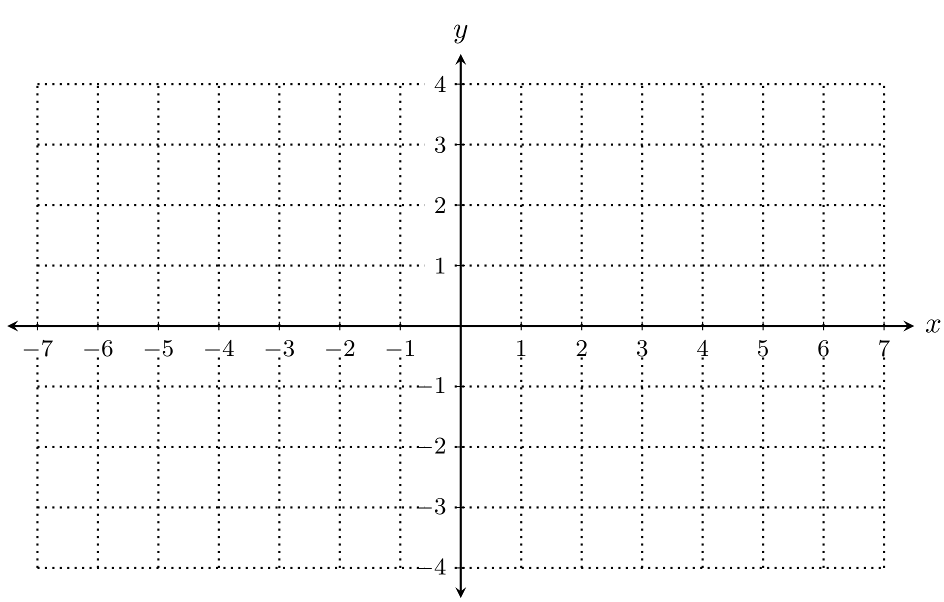
5. A particle moves along the x-axis so that its **velocity** at time t is given by $v(t) = 3t^2 - 14t + 6t^{1/2}$.

(a) At time $t = 4$, is the particle moving to the right or to the left? Explain.

(b) If the particle is at $x = 10$ when $t = 0$, what is the position of the particle at $t = 4$ seconds?

6. Sketch a graph of a function $y = f(x)$, whose domain is $(-\infty, \infty)$, that has all of the following properties:

- $\lim_{x \to -1^-} f(x) = 1$
- $\lim_{x \to -1^+} f(x) = 2$
- $\lim_{x \to -\infty} f(x) = 1$
- $f(-1) = 2$
- $\lim_{x \to 2} f(x) = 0$
- $\lim_{x \to \infty} f(x) = \infty$



7. This problem concerns the function $f(x) = x^3 + 6x^2 - 36x - 9$.

(a) Find the intervals on which $f(x)$ increases and the intervals on which it decreases.

(b) Locate any local extrema of $f(x)$ (i.e., give the x value and say if there is a local max or min there).

(c) Find the intervals on which $f(x)$ is concave up, and the intervals on which it is concave down.

8. Find the equation of the line tangent to the curve $f(x) = 3\sqrt[3]{x^2}$ at the point $(8, 12)$. Show all work.

9. Find the following integrals. If you make a substitution, clearly state u and du.

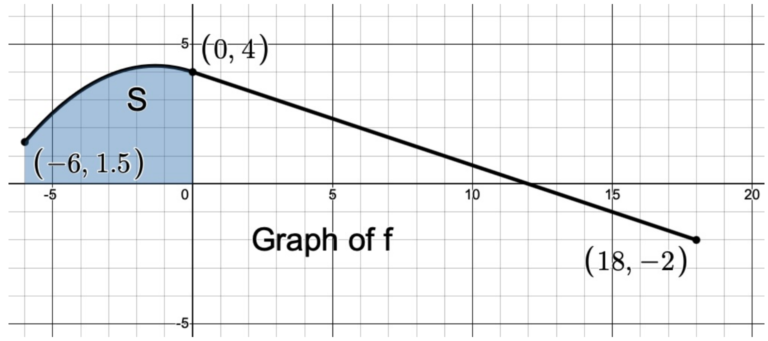
(a) $\int_{-2}^{1} \left(9x^2 - 4x + 1\right)\, dx$

(b) $\int \frac{8x+6}{\sqrt{4x^2+6x-9}}\, dx$

(c) $\int \frac{9x-1}{9x^2-2x+5}\, dx$

(d) $\int \frac{\cos(x)}{1+\sin^2(x)}\, dx$

10. The graph of a differentiable function f is shown. This function has a horizontal tangent at $x = -\frac{4}{3}$ and is linear for $0 \le x \le 18$. The shaded region S has an area of 21 square units.



Suppose g is the function defined as $g(x) = \int_0^x f(t)\,dt$

(a) Find $g(-6)$.

(b) Find $g(18)$.

(c) Does the function g have a local minimum in the interval $(-6, 18)$? Explain.

11. Suppose you need to enclose a rectangular region with a fence. The cost of the fencing for the north and south sides is \$9 per foot, and the cost of the east and west sides is \$6 per foot. You have \$504 to spend on fencing. Find the dimensions of the rectangle that maximize the enclosed area.

12. Find the derivative of the function $y = \int_3^{\cos(x)} \sqrt{1+t^2}\, dt$