MATH 200 – FINAL EXAM
R. Hammack December 11, 2025

Name: Richard

This is a closed-notes, closed book exam. No calculators, no computers, etc. Put phones away.
Answer the questions in the space provided, sowing work. Put your final answer in a $\boxed{\text{box}}$ when appropriate.

1. Find the limits using any appropriate method. Give an answer of ∞ or $-\infty$ if necessary.

(a) $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{x-1}$ $= \displaystyle\lim_{x\to 1} \frac{\frac{1}{x}}{1-0} = \frac{\frac{1}{1}}{1} = \boxed{1}$

form $\frac{0}{0}$

positive

(b) $\displaystyle\lim_{x\to 2} \frac{3}{(x-2)^2}$ $= \boxed{\infty}$

denominator approaches 0
and is positive

2. Find the derivatives of the functions below. You do NOT need to simplify your answer.

(a) $y = \dfrac{6-x^3}{x+5}$

$$y' = \frac{-3x^2(x+5)-(6-x^3)\cdot 1}{(x+5)^2} = \frac{-3x^3-15x^2-6+x^3}{(x+5)^2}$$

$$= \boxed{\frac{-2x^3-15x^2-6}{(x+5)^2}}$$

(b) $y = e^x - x^e + e^e$ $\quad y' = e^x - ex^{e-1} + 0 = \boxed{e^x - ex^{e-1}}$

(c) $y = \sin^{-1}(x)\cdot\sec(x)$ $\quad y' = \boxed{\frac{1}{\sqrt{1-x^2}}\sec(x) + \sin^{-1}(x)\sec(x)\tan(x)}$

(d) $y = \left(\ln(x^2+4)\right)^3$

$$y' = 3\left(\ln(x^2+4)\right)^2 D_x\left[\ln(x^2+4)\right]$$

$$= 3\left(\ln(x^2+4)\right)^2 \frac{2x}{x^2+4}$$

$$= \boxed{\frac{6x\left(\ln(x^2+4)\right)^2}{x^2+4}}$$

3. Given the equation $6x^2 + \ln(y) = 5 + 3y^2$, find y'

$D_x[6x^2 + \ln(y)] = D_x[5 + 3y^2]$ → $\boxed{y' = \dfrac{-12x}{\frac{1}{y} - 6y}}$

$12x + \dfrac{y'}{y} = 0 + 6yy'$

$\dfrac{y'}{y} - 6yy' = -12x$ $\quad = \dfrac{-12x}{\frac{1-6y^2}{y}}$

$y'\left(\frac{1}{y} - 6y\right) = -12x$

OR $\boxed{y' = \dfrac{12xy}{6y^2 - 1}}$

4. Information about two functions f and g is given in the table below.

x	0	3	5	8
$f(x)$	9	6	5	4
$f'(x)$	2	−7	4	7
$g(x)$	2	3	0	0
$g'(x)$	6	5	3	9

(a) Suppose $h(x) = \dfrac{f(x)}{g(x)}$. Find $h'(0)$.

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(0) = \frac{f'(0)\,g(0) - f(0)\,g'(0)}{(g(0))^2} = \frac{2\cdot 2 - 9\cdot 6}{2^2} = \frac{4-54}{4} = \frac{-50}{4} = \boxed{\frac{-25}{2}}$$

(b) Let k be a function for which $k'(x) = f(g(x))$. Is the graph of $k(x)$ concave up or down at $x = 5$?

Look at $k''(x) = f'(g(x))\,g'(x)$

Therefore $k''(5) = f'(g(5))\,g'(5) = f'(0)\cdot 3$

$= 2\cdot 3 = 6$

Because $k''(5) = 6 > 0$, the graph of $k(x)$ is $\boxed{\text{concave up at } x = 5}$

5. A particle moves along the x-axis so that its **velocity** at time t is given by $v(t) = 3t^2 - 14t + 6t^{1/2}$.

(a) At time $t = 4$, is the particle moving to the right or to the left? Explain.

At this time the velocity is $v(4) = 3 \cdot 4^2 - 14 \cdot 4 + 6\sqrt{4}$ $= 48 - 56 + 12 = 4$. Because the velocity is positive, the particle is moving to the $\boxed{\text{right}}$

(b) If the particle is at $x = 10$ when $t = 0$, what is the position of the particle at $t = 4$ seconds?

Position $= s(t) = \int v(t)\,dt = \int 3t^2 - 14t + 6t^{1/2}\,dt$

$$= t^3 - 7t^2 + 6\,\frac{t^{3/2}}{3/2} + c$$

6
7
12

$$= t^3 - 7t^2 + 4\sqrt{t}^{\,3} + c$$

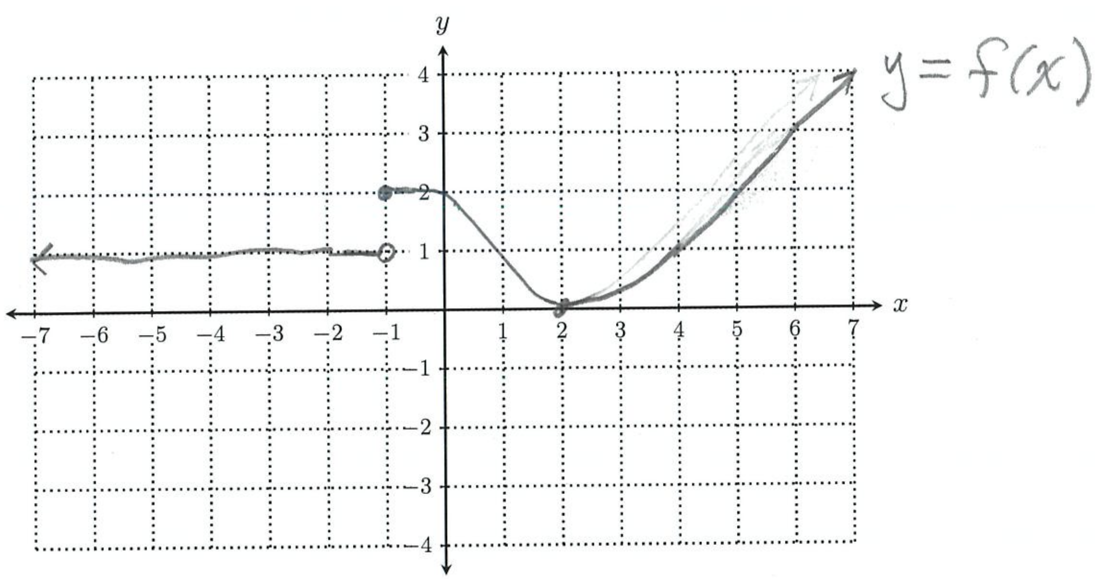
Know $10 = s(0) = 0^3 - 7 \cdot 0^2 + 4\sqrt{0}^{\,3} + c \Rightarrow c = 10$

Thus $\boxed{s(t) = t^3 + 7t^2 - 4\sqrt{t}^{\,3} + 10}$

Position at $t=4$ is $s(4) = 4^3 - 7 \cdot 4^2 + 4\sqrt{4}^{\,3} + 10 = 64 - 112 + 32 + 10 = \boxed{-6}$

6. Sketch a graph of a function $y = f(x)$, whose domain is $(-\infty, \infty)$, that has all of the following properties:

- $\lim_{x \to -1^-} f(x) = 1$
- $f(-1) = 2$
- $\lim_{x \to -1^+} f(x) = 2$
- $\lim_{x \to 2} f(x) = 0$
- $\lim_{x \to -\infty} f(x) = 1$
- $\lim_{x \to \infty} f(x) = \infty$



7. This problem concerns the function $f(x) = x^3 + 6x^2 - 36x - 9$.

(a) Find the intervals on which $f(x)$ increases and the intervals on which it decreases.

$$f'(x) = 3x^2 + 12x - 36 = 3(x^2 + 4x - 12)$$
$$= 3(x+6)(x-2)$$

critical points are -6 and 2

-6 $\quad$ 2

$+ + + \mid - - - - \mid + +$ $\quad f'(x) = 3(x+6)(x-2)$

$f(x)$ increases on $(-\infty, -6) \cup (2, \infty)$
$f(x)$ decreases on $(-6, 2)$

(b) Locate any local extrema of $f(x)$ (i.e., give the x value and say if there is a local max or min there).

Local max at $x = -6$
Local min at $x = 2$

By first derivative test.

(c) Find the intervals on which $f(x)$ is concave up, and the intervals on which it is concave down.

$$f''(x) = 6x + 12 \implies x = -2$$

-2

$- - - \mid + + +$ $\quad f''(x)$

f is concave up on $(-2, \infty)$
f is concave down on $(-\infty, -2)$

8. Find the equation of the line tangent to the curve $f(x) = 3\sqrt[3]{x^2}$ at the point $(8, 12)$. Show all work.

$$f(x) = 3x^{2/3}$$
$$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} = \frac{2}{\sqrt[3]{x}}$$

$$\text{slope} = m = f'(8) = \frac{2}{\sqrt[3]{8}} = 1$$

$$y - y_0 = m(x - x_0)$$
$$y - 12 = 1(x - 8)$$
$$\boxed{y = x + 4}$$

9. Find the following integrals. If you make a substitution, clearly state u and du.

(a) $\int_{-2}^{1} (9x^2 - 4x + 1)\, dx = \left[3x^3 - 2x^2 + x\right]_{-2}^{1}$

$= (3\cdot 1^3 - 2\cdot 1^2 + 1) - (3(-2)^3 - 2(-2)^2 + (-2))$

$= (3 - 2 + 1) - (-24 - 8 - 2) = 2 - (-34) = \boxed{36}$

(b) $\int \frac{8x+6}{\sqrt{4x^2+6x-9}}\, dx = \int \frac{1}{\sqrt{u}}\, du = \int u^{-1/2}\, du = \frac{u^{-1/2+1}}{-1/2+1} + C$

$u = 4x^2 + 6x - 9$

$\frac{du}{dx} = 8x + 6$

$du = (8x+6)\,dx$

$= \frac{u^{1/2}}{1/2} + C = 2\sqrt{u} + C$

$= \boxed{2\sqrt{4x^2+6x-9} + C}$

(c) $\int \frac{9x-1}{9x^2-2x+5}\, dx = \int \frac{1}{u}\frac{1}{2}\, du = \frac{1}{2}\ln|u| + C$

$u = 9x^2 - 2x + 5$

$\frac{du}{dx} = 18x - 2$

$\frac{1}{2}du = (9x-1)\,dx$

$= \boxed{\frac{1}{2}\ln|9x^2-2x+5| + C}$

(d) $\int \frac{\cos(x)}{1+\sin^2(x)}\, dx = \int \frac{1}{1+u^2}\, du = \tan^{-1}(u) + C$

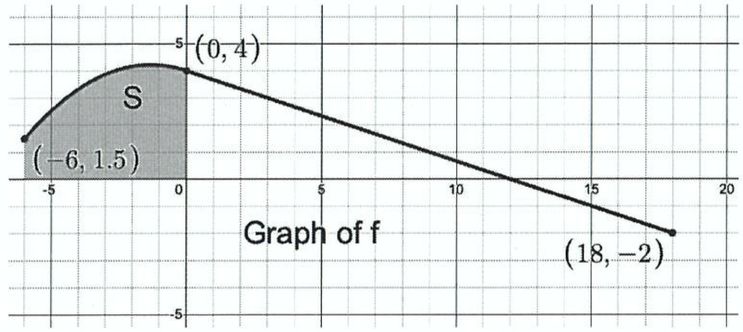
$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= \boxed{\tan^{-1}(\sin(x)) + C}$

10. The graph of a differentiable function f is shown. This function has a horizontal tangent at $x = -\frac{4}{3}$ and is linear for $0 \le x \le 18$. The shaded region S has an area of 21 square units.



Suppose g is the function defined as $g(x) = \int_0^x f(t)\,dt$

(a) Find $g(-6)$.

$$g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^{0} f(t)\,dt = \boxed{-21}$$

(b) Find $g(18)$.

$$g(18) = \int_0^{18} f(t)\,dt = A_{up} - A_{down}$$

$$= \frac{1}{2}12 \cdot 4 - \frac{1}{2}6 \cdot 2$$

$$= 24 - 6 = \boxed{18}$$

(c) Does the function g have a local minimum in the interval $(-6, 18)$? Explain.

$$g'(x) = D_x\left[\int_0^x f(t)\,dt\right] = f(x)$$

Now, $g'(12) = f(12) = 0$, so $x = 12$ is a critical point. Bcause $g'(x) = f(x)$ changes from + to − at $x = 12$, function $g(x)$ has a local max at $x = 12$, but $\boxed{\text{no local minimum}}$

11. Suppose you need to enclose a rectangular region with a fence. The cost of the fencing for the north and south sides is \$9 per foot, and the cost of the east and west sides is \$6 per foot. You have \$504 to spend on fencing. Find the dimensions of the rectangle that maximize the enclosed area.

Maximize area $= xy$

$\text{Area} = xy = x\,\frac{1}{2}(84-3x)$

$\text{Area} = A(x) = \frac{1}{2}(84x - 3x^2)$

maximize this on $(0, 28)$

$A'(x) = \frac{1}{2}(84-6x)$

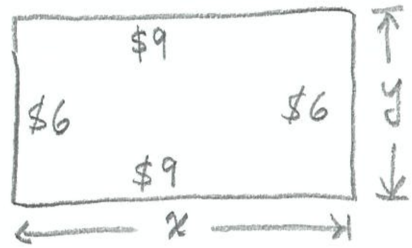
$= 42 - 3x = 0$

$-3x = -42$

$x = 14 \leftarrow$ critical point

$A''(x) = -3 < 0$, so $A(x)$ has a local maximum at $x = 14$.
There is only one critical point, so this is a global maximum.

Use $x = 14$, $y = \frac{1}{2}(84 - 3\cdot 14) = 42 - 21 = 21$



Constraint

$6\cdot 2y + 9\cdot 2x = 504$

$12y + 18x = 504$

$2y + 3x = 84$

$2y = 84 - 3x$

$y = \frac{1}{2}(84-3x)$

Note: x must obey

$3x < 84$

$x < \frac{84}{3} = 28$

Answer To maximize area, use $x = 16$ and $y = 18$

12. Find the derivative of the function $y = \int_3^{\cos(x)} \sqrt{1+t^2}\, dt$

$$\begin{cases} y = \int_3^u \sqrt{1+t^2}\, dt \\ u = \cos(x) \end{cases}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \sqrt{1+u^2}\cdot(-\sin(x)) = \boxed{-\sin(x)\sqrt{1+\cos^2(x)}}$$

chain rule

FTC I