Name: ______________________________ | MIDTERM EXAM ♡ | MATH 200 October 26, 2022

1. (35 pts.) Evaluate the following limits. Show steps, as appropriate.

(a) $\lim_{x\to 0} \dfrac{\sin(x)}{x} =$

(b) $\lim_{x\to\infty} \tan^{-1}\left(1+\dfrac{1}{x}\right) =$

(c) $\lim_{x\to\infty} e^{1/x} =$

(d) $\lim_{x\to 1} \dfrac{x^2-4x+3}{x^2+4x-5} =$

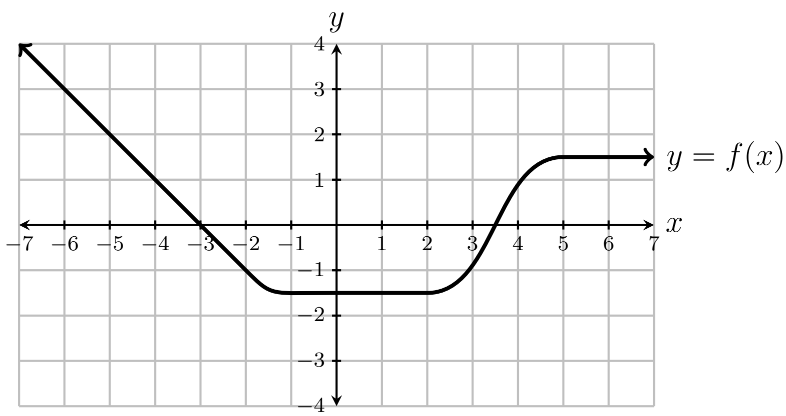
(e) $\lim_{h\to 0} \dfrac{\frac{1}{2+h}-\frac{1}{2}}{h} =$

(f) $\lim_{x\to 4^+} \dfrac{(-x+4)(x+2)}{|-x+4|} =$

(g) $\lim_{x\to 4^+} \dfrac{(-x+5)(x+2)}{|-x+4|} =$

2. (5 pts.) Use a limit definition of a derivative to find the derivative of $f(x) = 2 - 3x^2$.

3. (5 pts.) The graph of a function $f(x)$ is shown. Using the same grid, sketch the graph of $f'(x)$.



4. (5 pts.) Find all points (x, y) on the graph of $y = x + \dfrac{1}{x-3}$ where the tangent line is horizontal.

5. (30 pts.) Find the indicated derivatives.

(a) $f(\theta) = 5 + \ln(\pi\theta) + \sqrt{\theta^3}$

$f'(\theta) =$

$f''(\theta) =$

(b) $D_x\left[\dfrac{x}{x^3 + x^2 + 1}\right] =$

(c) $D_x\left[\, e^{4x}\sqrt{3x+1}\,\right] =$

(d) $D_x\left[\, \ln\left(\sec(x^3)\right)\,\right] =$

(e) $D_x\left[\, \tan^{-1}\left(\pi x\right)\,\right] =$

6. (5 pts.) Consider the equation $x\sin(y) = y^3$. Use implicit differentiation to find $\frac{dy}{dx}$.

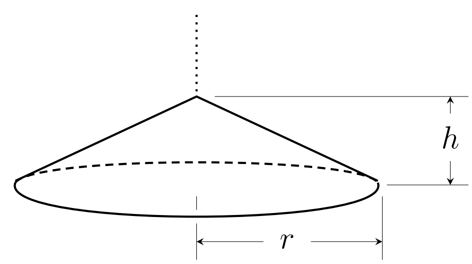
7. (5 pts.) Use logarithmic differentiation to find the derivative of $f(x) = (1 + 2x)^x$.

8. (10 pts.) A rock is thrown from a tower at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 48 + 32t - 16t^2$ feet. Please show your work in answering the following questions.

   (a) When does the rock hit the ground?

   (b) What is its velocity when it hits the ground?

9. (**Bonus:** 5 pts.) Sand falls at a rate of 6 cubic feet per minute, making a conical pile whose height h is always half its radius r. Find the rate of change of the radius r (in feet/min) when $r = 2$ feet.



Geometry formula: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.