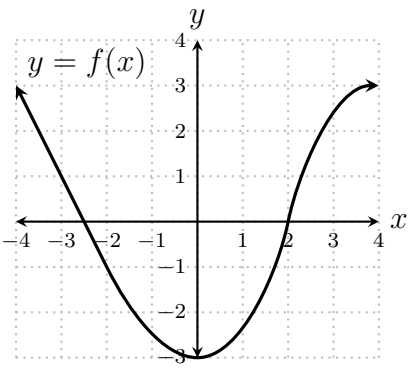
Name: _____ MATH 200 MIDTERM EXAM OCT. 17, 2024

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (10 points) Answer the questions about the function f graphed below.



(a) $\lim_{x\to\infty} f\left(\frac{1}{x}\right) =$

(b) $\lim_{h\to 0} \frac{f(-3+h)-f(-3)}{h} =$

(c) $\lim_{x\to -2} \frac{f(x)}{(1+f(x))^2} =$

(d) $\lim_{x\to 2} \frac{\sin\big(f(x)\big)+1}{f(x)+1} =$

(e) $\lim_{x\to 2} \frac{\sin\big(f(x)\big)}{f(x)} =$

2. (20 points) Find the limits

(a) $\lim_{x\to 0^+} \sin^{-1}(x-1) =$

(b) $\lim_{x\to e} 5\ln\left(x^3\right) =$

(c) $\lim_{x\to 3} \frac{x-3}{x^2-7x+12} =$

(d) $\lim_{x\to 1} \frac{\frac{1}{x}-1}{x-1} =$

3. (7 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \sqrt{1-x}$.

4. (7 points) An object moving on a straight line is $s(t) = t^3 - 3t^2$ feet from its starting point at time t seconds. Find its velocity when its acceleration is 12 feet per second per second.

5. (7 points) Suppose $f(x) = x^2 + 2x^3$ and $g(x) = x^2 - 2x^3 + 48x$. Find all x for which the tangent to $y = f(x)$ at $(x, f(x))$ is parallel to the tangent to $y = g(x)$ at $(x, g(x))$.

6. (35 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = \dfrac{\sqrt{2}}{x} + \pi x$

(b) $f(x) = \cos(x)\sin(x)$

(c) $f(x) = \cos\big(\sin(x)\big)$

(d) $f(x) = \tan^{-1}(-x)$

(e) $f(x) = \ln\left(e^{x^2-3x} + x\right)$

(f) $f(x) = \dfrac{1}{x^2 + 5x - 7}$

(g) $f(x) = \sqrt{\dfrac{x+1}{x-1}}^{\,3}$

7. (7 points) Given the equation $\dfrac{x}{y} = y^5 + x,$ find y'.

8. (7 points) Find the derivative of $f(x) = x^{\ln(x)}$.