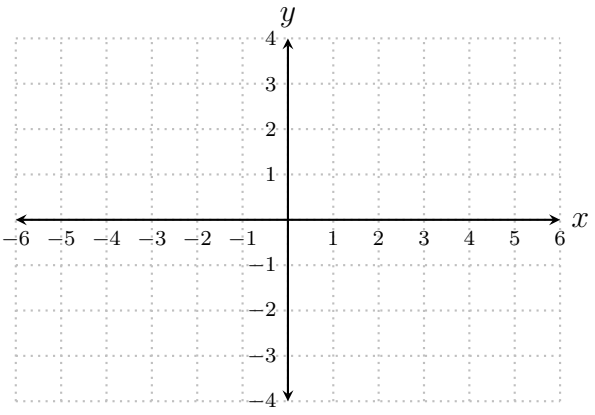
Name: _____ MATH 200 MIDTERM EXAM MARCH 17, 2022

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (10 points) Draw the graph of one function $f(x)$ meeting **all** of the following conditions.
   (a) $\lim_{x\to 3} f(x) = \infty$
   (b) $\lim_{x\to \infty} f(x) = \infty$
   (c) $\lim_{x\to -\infty} f(x) = 2$
   (d) f is continuous on $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$.
   (e) $f(1) = 1$
   (f) $f'(1) = 0$
   (g) $f'(-1)$ does not exist
   (h) $\lim_{x\to -2^+} f(x) = 1$
   (i) $\lim_{x\to -2^-} f(x) = 3$



2. (24 points) Find the limits.
   (a) $\lim_{x\to \infty} \tan^{-1}(x) =$
   (b) $\lim_{x\to 1/2} \sin^{-1}(x) =$
   (c) $\lim_{z\to 0} \dfrac{e^z - e^0}{z - 0} =$
   (d) $\lim_{x\to 2} \dfrac{\frac{4}{x} - 1}{x - 4} =$
   (e) $\lim_{x\to 4} \dfrac{\frac{4}{x} - 1}{x - 4} =$
   (f) $\lim_{x\to \infty} \dfrac{\frac{4}{x} - 1}{x - 4} =$

3. (6 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \sqrt{x}$.

4. (6 points) Find all x for which the tangent to the graph of $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + 1$ has slope 10.

5. (6 points) Suppose it costs $C(x)$ dollars to build a transmitting tower that is x meters high. Suppose it happens that $C'(100) = 1000$. Explain in simple terms what this means.

6. (35 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = x^3 + \pi^3$

(b) $f(x) = \dfrac{4}{\sqrt[3]{x}}$

(c) $f(x) = \cos\left(\dfrac{x+1}{x-1}\right)$

(d) $f(x) = \ln|x| \cdot \sec(x)$

(e) $f(x) = \left(\sin^{-1}(x)\right)^3$

(f) $f(x) = \dfrac{1}{x^2+1}$

(g) $y = x \ln\left(\sec\left(x^3 + x\right)\right)$

7. (7 points) Given the equation $y\ln(x) + y^2 = 5x$, find y'.

8. (6 points) A spherical balloon is inflated at a rate of 100π cubic feet per minute. How fast is the radius increasing at the instant the radius is 5 feet?

(A sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ cubic units, and surface area $S = 4\pi r^2$ square units.)