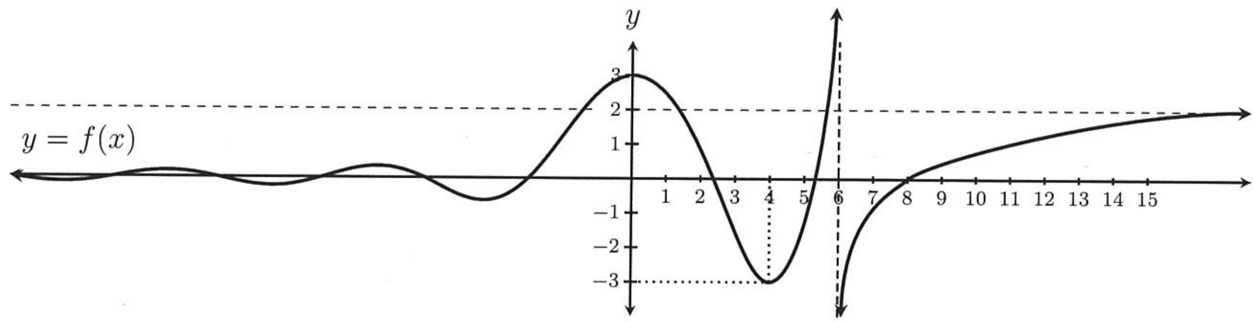
Name: Richard

Test 1

MATH 200
September 12, 2025

1. Answer the following questions about the function $y = f(x)$ graphed below.



(a) $\lim_{x\to 4} \dfrac{1}{3+f(x)} = \boxed{\infty}$

denominator approaches 0, pos.

(b) $\lim_{x\to 6} \dfrac{1}{f(x)} = \boxed{0}$

(c) $\lim_{x\to -\infty} f(x) = \boxed{0}$

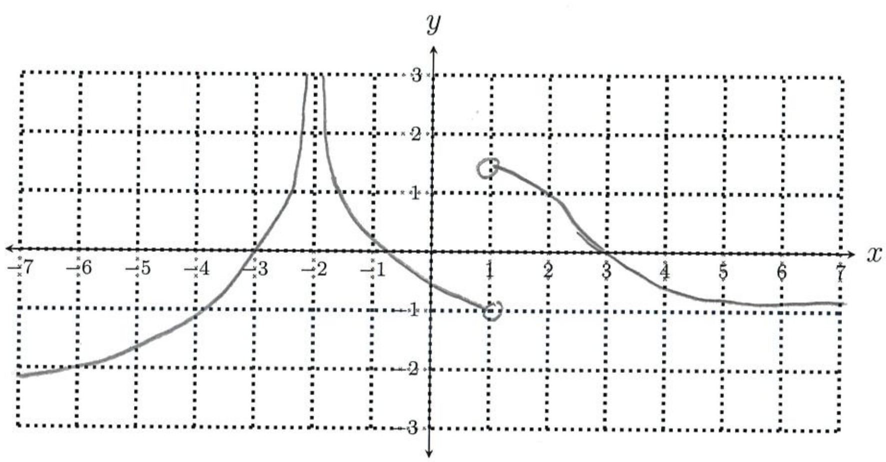
(d) $\lim_{x\to \infty} \cos\left(\dfrac{6\pi}{f(x)}\right) = \cos\left(\lim_{x\to\infty} \dfrac{6\pi}{f(x)}\right)$

$= \cos\left(\dfrac{6\pi}{2}\right) = \cos(3\pi) = \boxed{-1}$

(e) $\lim_{x\to 8^-} \dfrac{1}{f(x)} = \boxed{-\infty}$

2. Draw the graph of a function f that is continuous on $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$ and meets the following conditions.

- $\lim_{x\to -2} f(x) = \infty$
- $\lim_{x\to 3} f(x) = 0$
- $\lim_{x\to \infty} f(x) = -1$
- $\lim_{x\to 1^-} f(x) = -1$
- $\lim_{x\to 1^+} f(x) = \dfrac{3}{2}$



3. State the interval(s) on which the function $f(x) = \sqrt{\tan^{-1}(x)}$ is continuous.

$\boxed{[0,\infty)}$ ← Because $\tan^{-1}(x)$ is continuous and positive on $[0,\infty)$, so this is the domain of a function f built up from other continuous functions.

4. $\displaystyle\lim_{x\to 1} \frac{2-\frac{2}{x}}{x-1} =$ $\displaystyle\lim_{x\to 1} \frac{2-\frac{2}{x}}{x-1}\cdot\frac{x}{x} = \lim_{x\to 1}\frac{2x-2}{(x-1)x}$

$\displaystyle= \lim_{x\to 1}\frac{2\cancel{(x-1)}}{\cancel{(x-1)}x} = \lim_{x\to 1}\frac{2}{x} = \frac{2}{1} = \boxed{2}$

5. $\displaystyle\lim_{x\to 2} \sin^{-1}\left(\frac{x^2-3x+2}{x^2-2x}\right) =$ $\displaystyle\sin^{-1}\left(\lim_{x\to 2}\frac{x^3-3x+2}{x^2-2x}\right) = \sin^{-1}\left(\lim_{x\to 2}\frac{(x-1)\cancel{(x-2)}}{x\cancel{(x-2)}}\right)$

$\displaystyle= \sin^{-1}\left(\lim_{x\to 2}\frac{x-1}{x}\right) = \sin^{-1}\left(\frac{2-1}{2}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

approaches -1

6. $\displaystyle\lim_{x\to 0^+} \frac{x^2-3x+2}{x^2-2x} =$ $\displaystyle\lim_{x\to 0^+}\frac{(x-1)\cancel{(x-2)}}{x\cancel{(x-2)}} = \lim_{x\to 0^+}\frac{x-1}{x} = \boxed{-\infty}$

approaches 0, positive

7. $\displaystyle\lim_{x\to\infty} \frac{x^2-3x+2}{2x-x^2} =$ $\displaystyle\lim_{x\to\infty}\frac{x^2-3x+2}{2x-x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1-\frac{3}{x}+\frac{2}{x^2}}{\frac{2}{x}-1}$

$\displaystyle= \frac{1-0+0}{0-1} = \boxed{-1}$

8. $\lim_{x\to 1} \frac{\sin(x-1)+x-1}{x-1} =$

$$\lim_{x\to 1}\left(\frac{\sin(x-1)}{x-1} + \frac{x-1}{x-1}\right) = \lim_{x\to 1}\left(\frac{\sin(x-1)}{x-1} + 1\right)$$

$$= 1 + 1 = \boxed{2}$$

9. Give an example of a function (defined by an algebraic expression) that has a horizontal asymptote of $y = 3$ and two vertical asymptotes, $x = -1$ and $x = 5$.

$$f(x) = \frac{3x^2}{(x+1)(x-5)}$$

10. Use a limit definition of the derivative to find the derivative of $f(x) = 2x^2 + 1$.

$$f'(x) = \lim_{z\to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z\to x} \frac{(2z^2+1)-(2x^2+1)}{z-x}$$

$$= \lim_{z\to x} \frac{2z^2+1 - 2x^2-1}{z-x}$$

$$= \lim_{z\to x} \frac{2(z^2-x^2)}{z-x}$$

$$= \lim_{z\to x} \frac{2(z+x)\cancel{(z-x)}}{\cancel{z-x}}$$

$$= \lim_{z\to x} 2(z+x) = 2(x+x) = 2(2x) = \boxed{4x}$$

$$\therefore \boxed{f'(x) = 4x}$$