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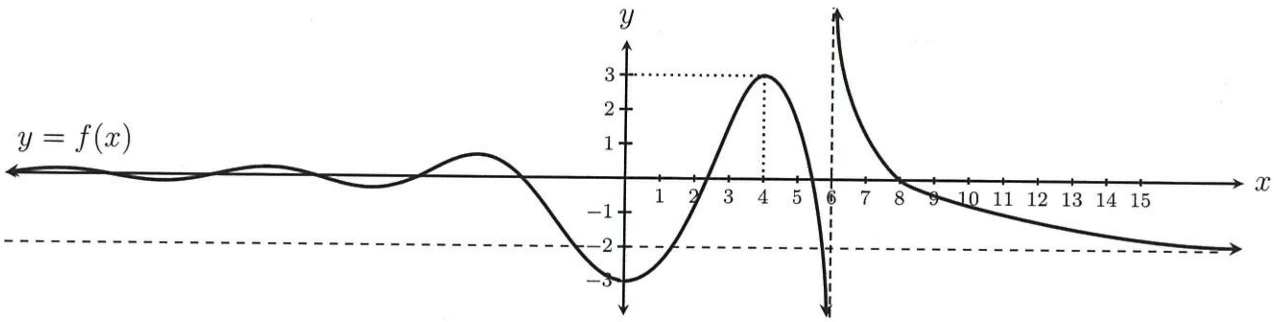
TEST 1

MATH 200
September 11, 2025

1. (8 points) Answer the following questions about the function $y = f(x)$ graphed below.



(a) $\lim_{x\to 4} \frac{1}{3+f(x)} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

(b) $\lim_{x\to 8} \frac{1}{(f(x))^2} = \boxed{\infty}$

(c) $\lim_{x\to\infty} f(x) = \boxed{-2}$

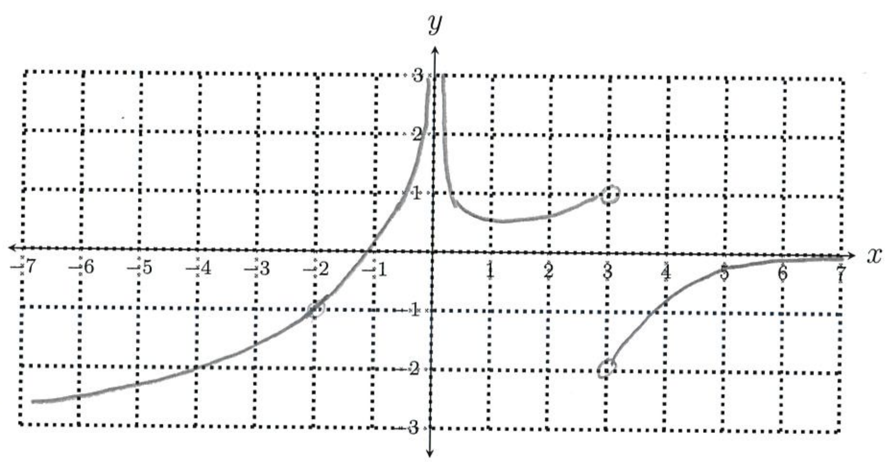
(d) $\lim_{x\to\infty} \sin\left(\frac{\pi}{f(x)}\right) = \sin\left(\lim_{x\to\infty} \frac{\pi}{f(x)}\right)$

$= \sin\left(-\frac{\pi}{2}\right) = \boxed{-1}$

(e) $\lim_{x\to 6} \frac{1}{f(x)} = \boxed{0}$

2. Draw the graph of a function f that is continuous on $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$ and meets the following conditions.

- $\lim_{x\to 0} f(x) = \infty$
- $\lim_{x\to -2} f(x) = -1$
- $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to 3^-} f(x) = 1$
- $\lim_{x\to 3^+} f(x) = -2$



3. State the interval(s) on which the function $f(x) = \frac{1}{\sqrt{4-x^2}}$ is continuous.

$\boxed{(2, 2)}$ ← Because $4-x^2$ is positive and non-zero on this interval, so this is the domain of a function built up from other continuous functions

4. $\lim_{x\to 1} \dfrac{\frac{4}{x} - 4}{1 - x} = \lim_{x\to 1} \dfrac{\frac{4}{x} - 4}{1-x} \cdot \dfrac{x}{x} = \lim_{x\to 1} \dfrac{4 - 4x}{(1-x)x}$

$= \lim_{x\to 1} \dfrac{4\cancel{(1-x)}}{\cancel{(1-x)}x} = \lim_{x\to 1} \dfrac{4}{x} = \dfrac{4}{1} = \boxed{4}$

5. $\lim_{x\to 2} \dfrac{x^2 - 2x}{x^2 - 5x + 6} = \lim_{x\to 2} \dfrac{x\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = \lim_{x\to 2} \dfrac{x}{x-3} = \dfrac{2}{2-3} = \boxed{-2}$

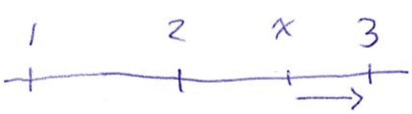
6. $\lim_{x\to\infty} \tan^{-1}\left(\dfrac{x^2 - 2x}{x^2 - 5x + 6}\right) = \tan^{-1}\left(\lim_{x\to\infty} \dfrac{x^2 - 2x}{x^2 - 5x + 6}\right)$

$= \tan^{-1}\left(\lim_{x\to\infty} \dfrac{x^2 - 2x}{x^2 - 5x + 6} \dfrac{1/x^2}{1/x^2}\right) = \tan^{-1}\left(\lim_{x\to\infty} \dfrac{1 - \frac{2}{x}}{1 - \frac{5}{x} + \frac{6}{x^2}}\right)$

$= \tan^{-1}\left(\dfrac{\cancel{1-0}}{\cancel{1-0+0}}\right) = \tan^{-1}(1) = \boxed{\dfrac{\pi}{4}}$

numerator approaches 3

7. $\lim_{x\to 3^-} \dfrac{x^2 - 2x}{x^2 - 5x + 6} = \lim_{x\to 3^-} \dfrac{x\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = \lim_{x\to 3^-} \dfrac{x}{x-3} = \boxed{-\infty}$



denominator approaches 0, negative

8. $\lim_{x\to 0} \frac{\sin(x) + x\cos(x)}{x} =$

$$= \lim_{x\to 0}\left(\frac{\sin(x)}{x} + \frac{x\cos(x)}{x}\right) = \lim_{x\to 0}\left(\frac{\sin(x)}{x} + \cos(x)\right)$$

$$= 1 + \cos(0) = 1+1 = \boxed{2}$$

9. Give an example of a function (defined by an algebraic expression) that has a horizontal asymptote of $y = -5$ and two vertical asymptotes, $x = 3$ and $x = 0$.

$$f(x) = \frac{-5x^2+1}{x(x-3)}$$

10. Use a limit definition of the derivative to find the derivative of $f(x) = \sqrt{2x}$.

$$f'(x) = \lim_{z\to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z\to x} \frac{\sqrt{2z} - \sqrt{2x}}{z - x}$$

$$= \lim_{z\to x} \frac{\sqrt{2z} - \sqrt{2x}}{z - x} \cdot \frac{\sqrt{2z} + \sqrt{2x}}{\sqrt{2z} + \sqrt{2x}}$$

$$= \lim_{z\to x} \frac{\sqrt{2z}^2 + \sqrt{2z}\sqrt{2x} - \sqrt{2x}\sqrt{2z} - \sqrt{2x}^2}{(z-x)(\sqrt{2z} + \sqrt{2x})}$$

$$= \lim_{x\to z} \frac{2z - 2x}{(z-x)(\sqrt{2z}+\sqrt{2x})}$$

$$= \lim_{x\to z} \frac{2\cancel{(z-x)}}{\cancel{(z-x)}(\sqrt{2z}+\sqrt{2x})} = \frac{2}{\sqrt{2x}+\sqrt{2x}}$$

$$= \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$$