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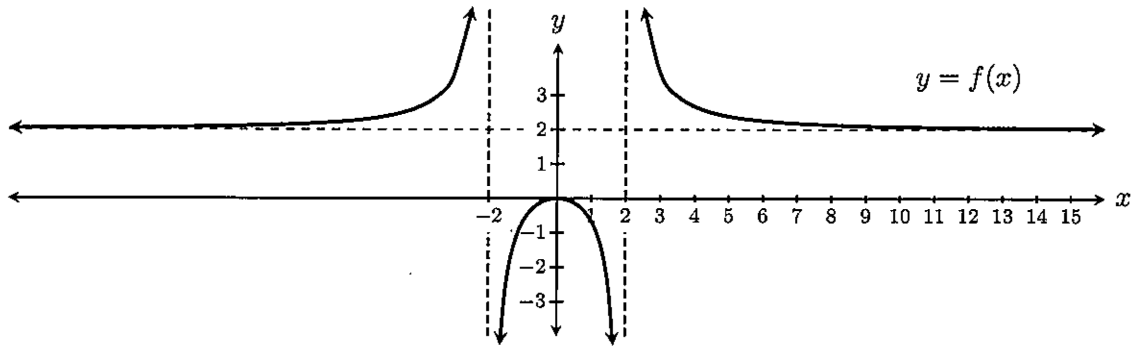
Test 1 ♣

MATH 200

February 6, 2026

Directions: Each problem is 5 points unless stated otherwise. Closed book, no calculators. Put phones away.

1. (6 points) Answer the following questions about the function $y = f(x)$ graphed below.



(a) $\lim_{x\to\infty} f(x) = \boxed{2}$

(b) $\lim_{x\to 2^-} f(x) = \boxed{-\infty}$

(c) $\lim_{x\to -\infty} \sin\left(\frac{\pi}{3f(x)}\right) = \sin\left(\lim_{x\to-\infty}\frac{\pi}{3f(x)}\right)$

$= \sin\left(\frac{\pi}{3\cdot 2}\right) = \sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$

(d) $\lim_{x\to 0} \cos^{-1}(f(x)) =$

$\cos^{-1}\left(\lim_{x\to 0} f(x)\right)$

$= \cos^{-1}(0) = \boxed{\frac{\pi}{2}}$

(e) $\lim_{x\to 2} \frac{1}{f(x)} = \boxed{0}$

(f) $\lim_{x\to 0} \frac{1}{f(x)} = \boxed{-\infty}$

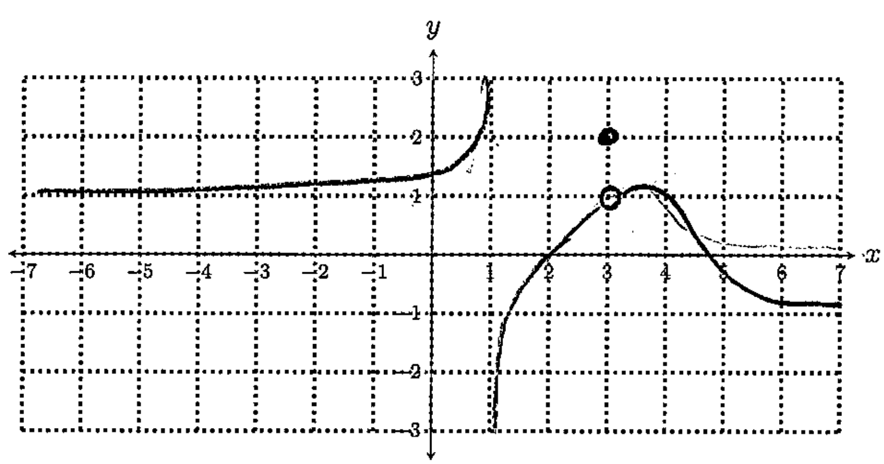
denominator approaches zero, negative

2. (4 points) The function $f(x)$ graphed in problem 1 above is a rational function. Give a possible algebraic expression for it.

$$f(x) = \frac{2x^2}{(x-2)(x+2)} = \frac{2x^2}{x^2-4}$$

3. (5 points) Draw the graph of a function that is continuous at all values of x **except** x=1 & x=3, *and* meets all of the following conditions.

- $\lim_{x\to 1^+} f(x) = -\infty$
- $\lim_{x\to 1^-} f(x) = \infty$
- $\lim_{x\to -\infty} f(x) = 1$
- $\lim_{x\to \infty} f(x) = -1$
- $\lim_{x\to 3} f(x) = 1$



4. State the intervals on which the function $f(x) = \dfrac{\ln(x)}{x^2 - x - 6}$ is continuous.

$$f(x) = \frac{\ln(x)}{(x+2)(x-3)}$$

Note: The domain of ln(x) is all positive numbers, so f(x) can't have negative numbers in its domain.

continuous on $\boxed{(0,3) \cup (3,\infty)}$

5. $\displaystyle \lim_{h\to 0} \frac{\frac{1}{7+h} - \frac{1}{7}}{h} = \lim_{h\to 0} \frac{\frac{1}{7+h} - \frac{1}{7}}{h} \cdot \frac{7(7+h)}{7(7+h)}$

$$= \lim_{h\to 0} \frac{7-(7+h)}{h\,7(7+h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\,7(7+h)} = \lim_{h\to 0} \frac{-1}{7(7+h)}$$

$$= \frac{-1}{7(7+0)} = \boxed{\frac{-1}{49}}$$

6. $\displaystyle \lim_{x\to 1} \frac{x^2-1}{8x^2-8x} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{8x\cancel{(x-1)}} = \lim_{x\to 1} \frac{x+1}{8x} = \frac{1+1}{8\cdot 1} = \boxed{\frac{1}{4}}$

7. $\displaystyle \lim_{x\to\infty} \left(\frac{x^2-3x+2}{8x-8x^2}\right)^{1/3} = \lim_{x\to\infty} \sqrt[3]{\frac{x^2-3x+2}{8x-8x^2}} = \sqrt[3]{\lim_{x\to\infty} \frac{x^2-3x+2}{8x-8x^2}}$

$$= \sqrt[3]{\lim_{x\to\infty} \frac{x^2-3x+2}{8x-8x^2} \cdot \frac{1/x^2}{1/x^2}} = \sqrt[3]{\lim_{x\to\infty} \frac{1 - \frac{3}{x} - \frac{2}{x^2}}{\frac{8}{x} - 8}}$$

$$= \sqrt[3]{\frac{1-0-0}{0-8}} = \sqrt[3]{\frac{-1}{8}} = \boxed{-\frac{1}{2}}$$

8. $\lim_{x\to\pi} \frac{\cos(x)}{1+\cos(x)} = \boxed{-\infty}$

Numerator approaches $\cos(\pi) = -1$.
Denominator approaches 0, and is positive

9. $\lim_{x\to 0} \frac{\cos(x)}{1+\cos(x)} =$ $\frac{\lim_{x\to 0} \cos(x)}{\lim_{x\to 0}(1+\cos(x))} = \frac{\cos(0)}{1+\cos(0)}$

$$= \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

10. Use a limit definition of the derivative to find the derivative of $f(x) = 2\sqrt{x}$.

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{2\sqrt{z}-2\sqrt{x}}{z-x}$$

$$= \lim_{z\to x} \frac{2(\sqrt{z}-\sqrt{x})}{\sqrt{z}^2-\sqrt{x}^2} = \lim_{z\to x} \frac{2\cancel{(\sqrt{z}-\sqrt{x})}}{(\sqrt{z}+\sqrt{x})\cancel{(\sqrt{z}-\sqrt{x})}}$$

$$= \lim_{z\to x} \frac{2}{\sqrt{z}+\sqrt{x}} = \frac{2}{\sqrt{x}+\sqrt{x}} = \frac{2}{2\sqrt{x}} = \boxed{\frac{1}{\sqrt{x}}}$$

Therefore $\boxed{f'(x) = \frac{1}{\sqrt{x}}}$