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**Test 2**

MATH 200
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1. $D_x\left[\, e^x + x^e + e^3 - x^3 + \ln(2) \,\right] = \boxed{e^x + e x^{e-1} - 3x^2}$

2. $D_x\left[\, x\sqrt{x^5 - x} \,\right] = 1 \cdot \sqrt{x^5 - x} + x \, \frac{1}{2}\left(x^5 - x\right)^{-\frac{1}{2}}\left(5x^4 - 1\right)$

$= \boxed{\sqrt{x^5 - x} + \frac{5x^5 - x}{2\sqrt{x^5 - x}}} = \frac{2\sqrt{x^5 - x}^{\,2}}{2\sqrt{x^5 - x}} + \frac{5x^5 - x}{2\sqrt{x^5 - x}} = \boxed{\frac{7x^5 - 3x}{2\sqrt{x^5 - x}}}$

3. $D_x\left[\, \left(\sin^{-1}(5x)\right)^3 \,\right] = 3\left(\sin^{-1}(5x)\right)^2 D_x\left[\sin^{-1}(5x)\right] = 3\left(\sin^{-1}(5x)\right)^2 \frac{1}{\sqrt{1 - (5x)^2}} \cdot 5$

$= \boxed{\frac{15\left(\sin^{-1}(5x)\right)^2}{\sqrt{1 - 25x^2}}}$

4. $D_x\left[\, \sec(x^2 + e^x) \,\right] = \boxed{\sec(x^2 + e^x)\tan(x^2 + e^x)(2x + e^x)}$

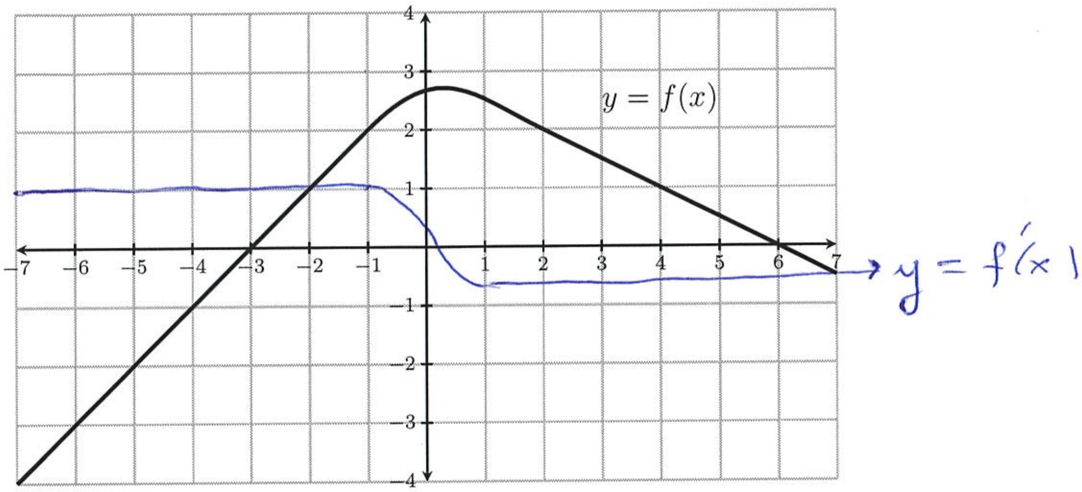
5. $D_x\left[\, e^{x/(x^2+1)} \,\right] = e^{\frac{x}{x^2+1}} \, \frac{1(x^2+1) - x(2x+0)}{(x^2+1)^2} = e^{\frac{x}{x^2+1}} \, \frac{x^2 + 1 - 2x^2}{(x^2+1)^2}$

$= \boxed{e^{\frac{x}{x^2+1}} \cdot \frac{1 - x^2}{(x^2+1)^2}}$

6. $D_w\left[\, \ln\left(w^3 - 4w^2 - 2w + 3\right) \,\right] = \boxed{\frac{3w^2 - 8w - 2}{w^3 - 4w^2 - 2w + 3}}$

7. The graph of a function $f(x)$ is shown below.

Using the same coordinate axis, sketch the graph of its derivative $f'(x)$




8. Given the equation $x^2 + y^3 = 3x^2y$, find $\frac{dy}{dx}$.

$$D_x[x^2 + y^3] = D_x[3x^2y]$$

$$2x + 3y^2\frac{dy}{dx} = 6xy + 3x^2\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 6xy - 2x$$

$$\frac{dy}{dx}\left(3y^2 - 3x^2\right) = 6xy - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{6xy - 2x}{3y^2 - 3x^2}}$$

9. Suppose it costs $C(x)$ dollars to drill a well to a depth of $x$ meters.

Suppose it happens that $C'(50) = 800$. Explain in simple terms what this means.

When you have drilled to a depth of 50 meters, cost is increasing at a rate of \$800 per meter.

∴ At a depth of 50 meters, you should expect to spend about \$800 to drill one extra meter, to a depth of 51 meters.

10. A spherical balloon is inflated at a rate of $64\pi$ cubic feet per minute.

How fast is the radius increasing at the instant the radius is 2 feet?

$$V = \frac{4}{3}\pi r^3$$

$$D_t[V] = D_t\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$64\pi = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{64\pi}{4\pi r^2} = \frac{16}{r^2}$$

Ans: $$\left.\frac{dr}{dt}\right|_{r=2} = \frac{16}{2^2} = \boxed{4 \text{ feet/min}}$$

(A sphere of radius $r$ has volume $V = \frac{4}{3}\pi r^3$ cubic units, and surface area $S = 4\pi r^2$ square units.)