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TEST 2

MATH 200
October 10, 2025

1. $D_x\Big[\cos(3x) + \ln(2) - x^\pi + e^{-x}\Big] = \boxed{-3\sin(3x) - \pi x^{\pi-1} - e^{-x}}$

2. $D_w\Big[\ln\left(w^3 - 4w^2 - 2w + 3\right)\Big] = \boxed{\dfrac{3w^2 - 8w - 2}{w^3 - 4w^2 - 2w + 3}}$

3. $D_x\left[\dfrac{x}{\sqrt{x^5 - x}}\right] = \dfrac{(1)\sqrt{x^5-x} - x\cdot\dfrac{5x^4-1}{2\sqrt{x^5-x}}}{\sqrt{x^5-x}^{\,2}} = \boxed{\dfrac{\sqrt{x^5-x} - \dfrac{5x^5-x}{2\sqrt{x^5-x}}}{x^5-x}}$

4. $D_x\Big[\left(x^2 + \tan^{-1}(5x)\right)^4\Big] = 4\left(x^2 + \tan^{-1}(5x)\right)^3\left(2x + \dfrac{1}{1+(5x)^2}\cdot 5\right)$

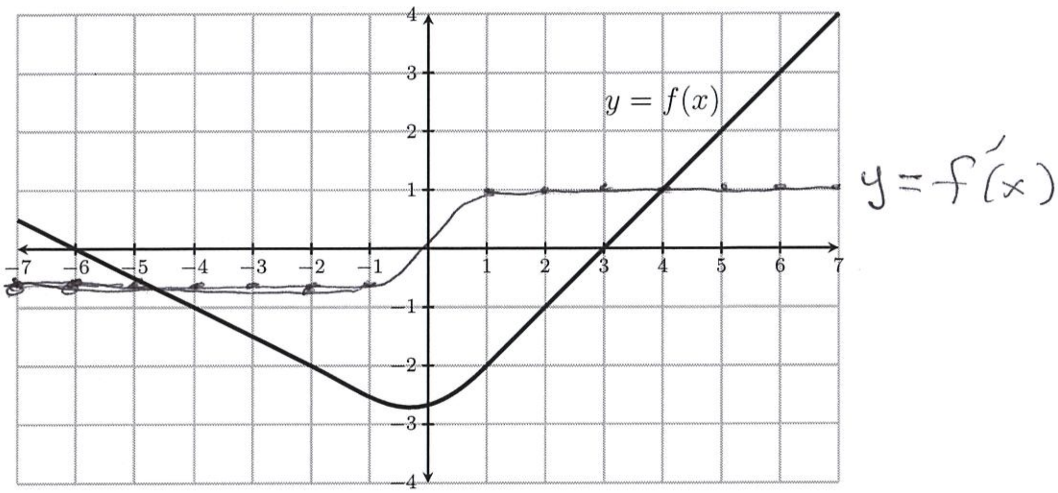
$= \boxed{4\left(x^2 + \tan^{-1}(5x)\right)^3\left(2x + \dfrac{5}{1+25x^2}\right)}$

5. $D_x\Big[\csc\left(\dfrac{\pi}{x}\right)\Big] = -\csc\left(\dfrac{\pi}{x}\right)\cot\left(\dfrac{\pi}{x}\right)D_x\left[\pi x^{-1}\right] = \boxed{\dfrac{\pi\csc\left(\frac{\pi}{x}\right)\cot\left(\frac{\pi}{x}\right)}{x^2}}$

6. $D_x\Big[\ln(x)e^{\tan(x^2)}\Big] = \boxed{\dfrac{1}{x}e^{\tan(x^2)} + \ln(x)\,e^{\tan(x^2)}\sec^2(x^2)\,2x}$

7. The graph of a function $f(x)$ is shown below.

Using the same coordinate axis, sketch the graph of its derivative $f'(x)$



8. Given the equation $y^2 + x^3 = 3xy^3$, find $\dfrac{dy}{dx}$.

$y = f(x)$

$$D_x\left[y^2 + x^3\right] = D_x\left[3xy^3\right]$$

$$2yy' + 3x^2 = 3y^3 + 3x \cdot 3y^2y'$$

$$2yy' - 9xy^2y' = 3y^3 - 3x^2$$

$$y'(2y - 9xy^2) = 3y^3 - 3x^2$$

$$\boxed{y' = \frac{3y^2 - 3x^2}{2y - 9xy^2}}$$

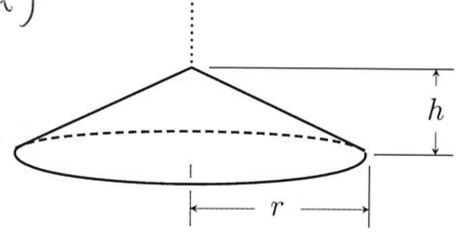
9. Suppose it costs $C(x)$ dollars to build a transmitting tower x meters high.
Suppose it happens that $C'(60) = 1020$. Explain in simple terms what this means.

When the tower is 60 meters high, the cost is increasing at a rate of \$1020 per meter. At this rate it will cost an additional \$1020 to build it an additional meter higher. (to 61 meters)

10. Sand falls at a rate of 4 cubic feet per minute, making a conical pile whose height h is always half its radius r. Find the rate of change of the radius r (in feet/min) when $r = 2$ feet.

Know: $\frac{dV}{dt} = 4$ (cubic ft / min)

Want: $\frac{dr}{dt}$ (ft/min) when $r=2$.



$V = \frac{1}{3}\pi r^2 h \leftarrow \{h = \frac{1}{2} r$

$V = \frac{1}{3}\pi r^2 \frac{r}{2}$

$V = \frac{\pi r^3}{6}$

$D_t[V] = D_t\left[\frac{\pi r^3}{6}\right]$

$\frac{dV}{dt} = \frac{\pi r^2}{2}\frac{dr}{dt}$

$4 = \frac{\pi r^2}{2}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{8}{\pi r^2}$

Ans $\left.\frac{dr}{dt}\right|_{r=2} = \frac{8}{\pi \cdot 2^2} = \boxed{\frac{2}{\pi}\ \text{feet/min}}$

Geometry formula: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.