

Directions: Each question is 5 points. Closed book, no calculators. Put phones away. Put your answer in a box.

1. Answer the questions about the functions graphed below.

(a) $g'(3) =$ 2

(b) $f'(3) =$ 0

(c) $\lim_{x \rightarrow -2} g'(x) =$ $-\infty$

(d) If $h(x) = f(g(x))$, then $h'(3) =$ -2

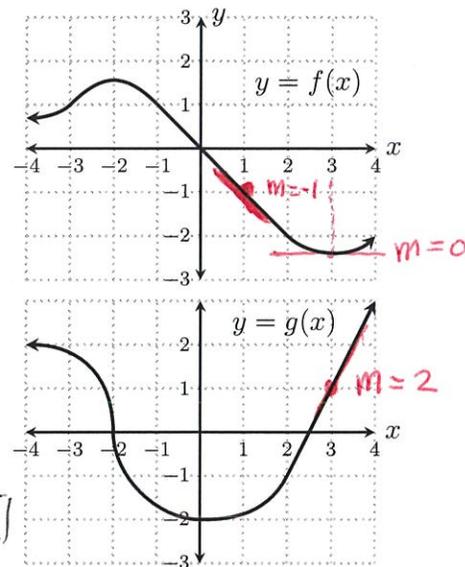
$$h'(x) = f'(g(x))g'(x)$$

$$h'(3) = f'(g(3))g'(3) = f'(1) \cdot 2 = (-1) \cdot 2 =$$
 -2

(e) If $h(x) = f(x)g(x)$, then $h'(0) =$ 2

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(0) = f'(0)g(0) + f(0)g'(0) = (-1)(-2) + (0)(0) =$$
 2



2. (8 points) Find the derivatives of the following functions.

(a) $f(x) = e^x + 3x^4 + \pi^2$ $f'(x) =$ $e^x + 12x^3$

(b) $f(x) = \sec^{-1}(x)$ $f'(x) =$ $\frac{1}{|x|\sqrt{x^2-1}}$

(c) $f(x) = e^{3x}$ $f'(x) =$ $3e^{3x}$

(d) $f(x) = \sin(\pi x)$ $f'(x) =$ $\pi \cos(\pi x)$

3. Find the equation of the tangent line to the graph of $y = \tan^{-1}(x)$ at the point where $x = 1$. Show work.

Point: $(1, \tan^{-1}(1)) = (1, \frac{\pi}{4}) = (x_0, y_0)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Slope: $m = \frac{dy}{dx}\bigg|_{x=1} = \frac{1}{1+1^2} = \frac{1}{2}$

Point-Slope Formula:

$$y - y_0 = m(x - x_0)$$

$$y - \frac{\pi}{4} = \frac{1}{2}(x - 1)$$

$$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$$

$$4. \frac{d}{dx} \left[\frac{1}{(3x+1)^7} \right] = \frac{d}{dx} \left[(3x+1)^{-7} \right] = -7(3x+1)^{-8} (3+0) = \boxed{\frac{-21}{(3x+1)^8}}$$

$$5. \frac{d}{dx} \left[\frac{x}{e^x} \right] = \frac{1 \cdot e^x - x e^x}{(e^x)^2} = \frac{e^x(1-x)}{e^x e^x} = \boxed{\frac{1-x}{e^x}}$$

$$6. \frac{d}{dx} \left[x^2 \sin(x^2) \right] = 2x \sin(x^2) + x^2 \cos(x^2) 2x$$

$$= \boxed{2x \sin(x^2) + 2x^3 \cos(x^2)}$$

$$7. \frac{d}{dx} \left[\sqrt{\ln(x^4 + x^2 + 1)} \right] = \frac{d}{dx} \left[\left(\ln |x^4 + x^2 + 1| \right)^{\frac{1}{2}} \right]$$

$$= \frac{1}{2} \left(\ln |x^4 + x^2 + 1| \right)^{-\frac{1}{2}} \frac{4x^3 + 2x}{x^4 + x^2 + 1} = \boxed{\frac{4x^3 + 2x}{2 \sqrt{\ln |x^4 + x^2 + 1|} (x^4 + x^2 + 1)}}$$

$$= \boxed{\frac{2x^3 + x}{(x^4 + x^2 + 1) \sqrt{\ln |x^4 + x^2 + 1|}}}$$

8. Suppose $y = \ln(x) - \frac{1}{x} + \frac{1}{2}$.

(a) $\frac{dy}{dx} = \boxed{\frac{1}{x} + \frac{1}{x^2}}$

(b) $\frac{d^2y}{dx^2} = \boxed{-\frac{1}{x^2} - \frac{2}{x^3}}$

9. Given the equation $x^4 + 2y + y^4 = \cos(x)$, find y' .

Show work.

$$D_x [x^4 + 2y + y^4] = D_x [\cos(x)]$$

$$4x^3 + 2y' + 4y^3 y' = -\sin(x)$$

$$2y' + 4y^3 y' = -\sin(x) - 4x^3$$

$$y'(2 + 4y^3) = -\sin(x) - 4x^3$$

$$y' = \frac{-\sin(x) - 4x^3}{2 + 4y^3}$$

10. A function $f(x)$ is graphed below. Sketch the graph of its derivative $f'(x)$.

