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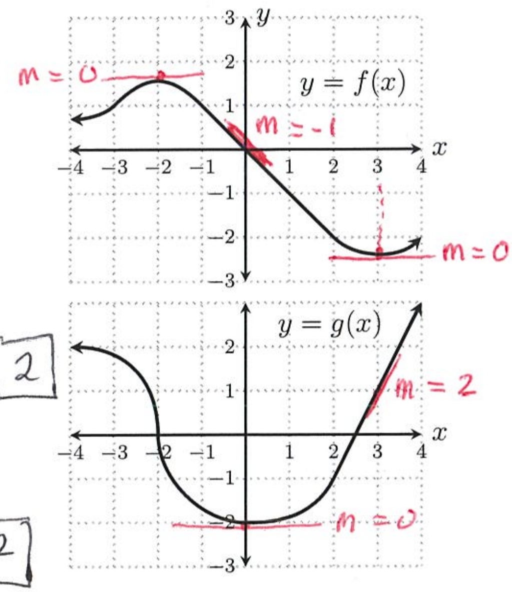
TEST 2 ♡

MATH 200
March 6, 2026

Directions: Each question is 5 points. Closed book, no calculators. Put phones away. Put your answer in a $\boxed{\text{box}}$.

1. Answer the questions about the functions graphed below.



   (a) $f'(-2) =$ $\boxed{0}$

   (b) $f'(0) =$ $\boxed{-1}$

   (c) $\lim_{x\to -2} g'(x) =$ $\boxed{-\infty}$

   (d) If $h(x) = f(x)g(x)$, then $h'(0) =$ $\boxed{2}$

   $h'(x) = f'(x)g(x) + f(x)g'(x)$

   $h'(0) = f'(0)g(0) + f(0)g'(0) = (-1)(-2) + (0)(0) = \boxed{2}$

   (e) If $h(x) = f(g(x))$, then $h'(3) =$ -2

   $h'(x) = f'(g(x))\,g'(x)$

   $h'(3) = f'(g(3))\,g'(3) = f'(1)\cdot 2 = (-1)(2) = \boxed{-2}$

2. (8 points) Find the derivatives of the following functions.

   (a) $f(x) = x^4 - 3x + \pi^2$ $\qquad f'(x) = 4x^3 - 3$

   (b) $f(x) = \sin^{-1}(x)$ $\qquad f'(x) = \frac{1}{\sqrt{1-x^2}}$

   (c) $f(x) = e^{-x}$ $\qquad f'(x) = -e^{-x}$

   (d) $f(x) = \tan(\pi x)$ $\qquad f'(x) = \pi \sec^2(\pi x)$

3. Find the equation of the tangent line to the graph of $y = \tan^{-1}(x)$ at the point where $x = 1$. Show work.

<u>Point</u>: $(1, \tan^{-1}(1)) = (1, \pi/4) = (x_0, y_0)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

<u>Slope</u>: $\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1+1^2} = \frac{1}{2} = m$

<u>Point-Slope formula</u>: $y - y_0 = m(x - x_0)$

$$y - \frac{\pi}{4} = \frac{1}{2}(x-1)$$

$$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$$

4. $\frac{d}{dx}\left[\sqrt{x^4+x^2+1}\right] = \frac{d}{dx}\left[(x^4+x^2+1)^{\frac{1}{2}}\right] = \frac{1}{2}(x^4+x^2+1)^{-\frac{1}{2}}(4x^3+2x)$

$$= \frac{4x^3+2x}{2\sqrt{x^4+x^2+1}} = \boxed{\frac{2x^3+x}{\sqrt{x^4+x^2+1}}}$$

5. $\frac{d}{dx}\left[x^2\cos(x^2)\right] = 2x\cos(x^2) + x^2(-\sin(x^2)2x)$

$$= \boxed{2x\cos(x^2) - 2x^3\sin(x^2)}$$

6. $\frac{d}{dx}\left[\frac{e^x}{x}\right] = \frac{e^x x - e^x \cdot 1}{x^2} = \boxed{\frac{e^x(x-1)}{x^2}}$

7. $\frac{d}{dx}\left[\left(x^3+e^{\sec(x)}\right)^7\right] = \boxed{7\left(x^3+e^{\sec(x)}\right)^6\left(3x^2+e^{\sec(x)}\sec(x)\tan(x)\right)}$

8. Suppose $y = x\ln(x) - x + 3$.

(a) $\frac{dy}{dx} = 1\cdot\ln(x) + x\left(\frac{1}{x}\right) - 1 + 0 = \ln(x) + 1 - 1 = \boxed{\ln(x)}$

(b) $\frac{d^2y}{dx^2} =.$ $\boxed{\frac{1}{x}}$

9. Given the equation $x^4 + 2xy + y^4 = \sin(x)$, find y'. Show work.

$$D_x\left[x^4 + 2xy + y^4\right] = D_x\left[\sin(x)\right]$$

$$4x^3 + 2y + 2xy' + 4y^3y' = \cos(x)$$

$$2xy' + 4y^3y' = \cos(x) - 4x^3 - 2y$$

$$y'(2x + 4y^3) = \cos(x) - 4x^3 - 2y$$

$$\boxed{y' = \frac{\cos(x) - 4x^3 - 2y}{2x + 4y^3}}$$

10. A function $f(x)$ is graphed below. Sketch the graph of its derivative $f'(x)$.