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TEST 3

MATH 200

November 7, 2025

1. Evaluate the limits.

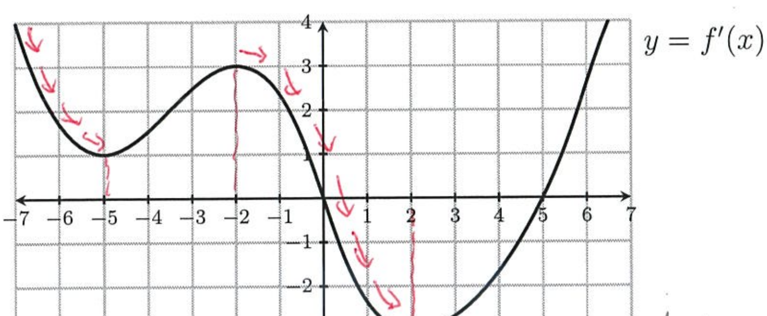
(a) $\lim\limits_{x\to\infty} 4xe^{-3x} = \lim\limits_{x\to\infty} \dfrac{4x}{e^{3x}} = \lim\limits_{x\to\infty} \dfrac{4}{3e^{3x}} = \boxed{0}$

form $\infty \cdot 0$ form $\frac{\infty}{\infty}$ denominator approaches ∞

(b) $\lim\limits_{x\to 0} \dfrac{x^2}{1-\cos(x)} = \lim\limits_{x\to 0} \dfrac{2x}{\sin(x)} = \lim\limits_{x\to 0} \dfrac{2}{\cos(x)} = \dfrac{2}{\cos(0)} = \dfrac{2}{1} = \boxed{2}$

form $\frac{0}{0}$ form $\frac{0}{0}$

2. The graph of **the derivative** $\underline{f'(x)}$ of a function $f(x)$ is shown. Answer the questions about $\underline{f(x)}$.



+ + + + + + + − − − − − + + $f'(x)$

− − | + + + | − − − − | + + + + + $f''(x)$

(a) What are the critical points of $f(x)$? $\boxed{x=0 \quad x=5}$ (because $f'(0)=0$ & $f'(5)=0$)

(b) On what intervals is $f(x)$ decreasing? $\boxed{(0,5)}$ (because $f'(x)<0$ on this interval)

(c) State the locations (x values) of any local minima of $f(x)$. $\boxed{x=5}$ (by 1st derivative test)

(d) State the locations (x values) of any local maxima of $f(x)$. $\boxed{x=0}$ (by 1st derivative test)

(e) State the locations (x values) of any inflection points of $f(x)$. $\boxed{x=-5, -2, 2}$

3. Find the absolute extrema of $f(x) = x^2(x-3)^4$ on $[2, 4]$.

$$f'(x) = 2x(x-3)^4 + x^2\,4(x-3)^3$$
$$= 2x(x-3)^3\big((x-3) + 2x\big)$$
$$= 2x(x-3)^3(3x-3)$$
$$= 6x(x-3)^3(x-1)$$

$\boxed{x=0 \qquad x=3 \qquad x=1}$ critical points.

Only $x=3$ is in the interval

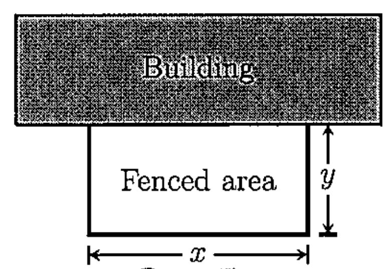
$f(2) = 2^2(2-3)^4 = 4\cdot(-1)^4 = 4$

$f(3) = 3^2(3-3)^4 = 9\cdot 0^4 = 0$ ← minimum

$f(4) = 4^2(4-3)^4 = 16\cdot 1^4 = 16$ ← maximum

$\boxed{\begin{array}{l}\text{Global maximum of } f(4) = 16 \text{ at } x=4\\ \text{Global minimum of } f(3) = 0 \text{ at } x=3\end{array}}$

4. You have 160 feet of fencing material to enclose a rectangular region. One side borders a building, so no fencing is required for that side. Find the dimensions x and y that maximize the fenced area.



Maximize : Area $= xy$

Constraint :

$x + 2y = 160$

$2y = 160 - x$

$y = \dfrac{160-x}{2}$

Area $= A(x) = x\,\dfrac{160-x}{2}$

$\boxed{A(x) = 80x - \tfrac{1}{2}x^2}$

Maximize this on $[0, 160)$

$A'(x) = 80 - x = 0$

$x = 80$ ← critical point

$A''(x) = -1$

$A''(80) = -1 < 0$ therefore a local maximum of $A(x)$ at $x = 80$, but there is only 1 critical point so this is a global maximum.

$x = 80$

$y = \dfrac{160-80}{2} = 40$

Answer To maximize area, use dimensions $x = 80$, $y = 40$

5. The questions on this page are about the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x + 1$.

(a) Find the intervals on which $f(x)$ increases and on which it decreases.

$f'(x) = x^2 - 8x + 12 = (x-2)(x-6) = 0$

$x = 2 \quad x = 6$ ← critical points

0 2 6

+ + + + | - - - - - - | + + + $f'(x)$

$\boxed{\begin{array}{l} f(x) \text{ increases on } (-\infty, 2) \cup (6, \infty) \\ f(x) \text{ decreases on } (2, 6) \end{array}}$

(b) Find and identify the local extrema. (Their x values will suffice.)

By first derivative test:

$\boxed{\begin{array}{l} f(x) \text{ has a local maximum at } x = 2 \\ f(x) \text{ has a local minimum at } x = 6 \end{array}}$

(c) Find the intervals on which $f(x)$ is concave up and on which it is concave down.

$f''(x) = 2x - 8 = 2(x-4)$

4

- - - - - - | + + + + $f''(x)$

$\boxed{\begin{array}{l} f(x) \text{ is concave up on } (4, \infty) \\ f(x) \text{ is concave down on } (-\infty, 4) \end{array}}$

(d) State the locations of all inflection points of $f(x)$. (Their x values will suffice.)

Inflection point at $x = 4$.

(e) Find and identify the global extrema of $f(x)$ on the interval $(1, 5)$.

There is only one critical point on this open interval and it is $x = 2$. By part (b), $f(x)$ has a local maximum at $x = 2$. But since this is the only critical point, the local max is a global max.

$\boxed{\text{Global max at } x = 2}$ (no global minimum)