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TEST 3

MATH 200
November 7, 2025

1. Evaluate the limits.

(a) $\lim_{x\to 0} \frac{e^{\sin(x)}-1}{x} = \lim_{x\to 0} \frac{e^{\sin(x)}\cos(x)}{1} = e^{\sin(0)}\cos(0) = e^{0}\cdot 1 = \boxed{1}$

form $\frac{0}{0}$

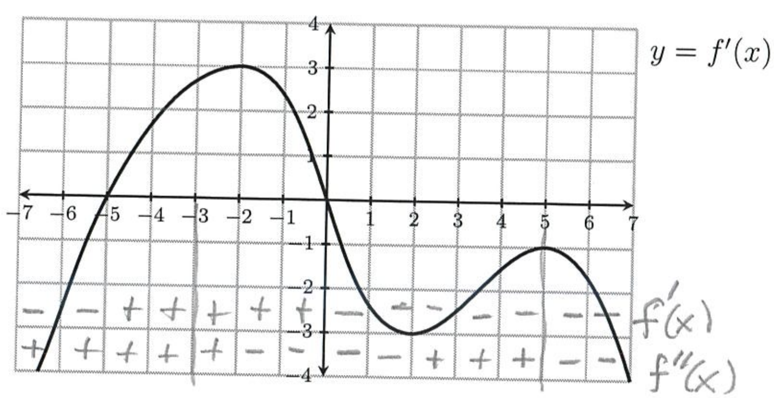
(b) $\lim_{x\to\infty} 3xe^{-2x} = \lim_{x\to\infty} \frac{3x}{e^{2x}} = \lim_{x\to\infty} \frac{3}{2e^{x}} = \boxed{0}$

form $\infty\cdot 0$

form $\frac{\infty}{\infty}$

denominator is approaching ∞

2. The graph of **the derivative** $f'(x)$ of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) What are the critical points of $f(x)$? $\boxed{x=-5,\ x=0}$ (because $f'(-5)=0$ & $f'(0)=0$)

(b) On what intervals is $f(x)$ decreasing? $\boxed{(-\infty,-5)\cup(0,\infty)}$ (that's where $f'(x)<0$)

(c) State the locations (x values) of any local minima of $f(x)$. $\boxed{x=-5}$ (by 1st derivative test)

(d) State the locations (x values) of any local maxima of $f(x)$. $\boxed{x=0}$ (by 1st derivative test

(e) State the locations (x values) of any inflection points of $f(x)$. $\boxed{x=-2, 2, 5}$

3. Find the absolute extrema of $f(x) = x^3(x-2)^3$ on $[1, 3]$.

$$f'(x) = 3x^2(x-2)^3 + x^3\, 3(x-2)^2$$
$$= 3x^2(x-2)^2((x-2)+x)$$
$$= 3x^2(x-2)^3(2x-2)$$

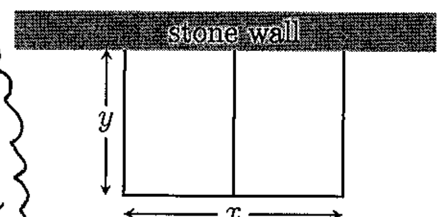
$x=0$ $\quad x=2$ $\quad x=1$ ← critical points

$x=1$ is an endpoint and $x=0$ is not in the interval

$$f(1) = 1^3(1-2)^3 = 1\cdot(-1) = -1 \quad \leftarrow \text{min}$$
$$f(2) = 2^3(2-2)^3 = 8\cdot 0 = 0$$
$$f(3) = 3^3(3-2)^3 = 27\cdot 1^3 = 27 \quad \leftarrow \text{max}$$

There is a global maximum of $f(3) = 27$ at $x=3$.
There is a global minimum of $f(1) = -1$ at $x=1$.

4. You have a 300 feet of chain link fence to enclose <u>two</u> rectangular pens formed along a stone wall, as illustrated. No fencing is needed along the stone wall. What dimensions (i.e. x feet by y feet) yield the greatest total enclosed area?



Area $= xy$

Constraint:
$$300 = x + 3y$$
$$3y = 300 - x$$
$$y = 100 - \tfrac{1}{3}x$$

Area $= A(x) = x\left(100 - \tfrac{1}{3}x\right)$

$A(x) = 100x - \frac{1}{3}x^2$
Maximize this on $(0, 300)$

$$A'(x) = 100 - \frac{2}{3}x = 0$$
$$300 - 2x = 0$$
$$x = \frac{300}{2} = 150 \leftarrow \text{critical point.}$$

$A''(x) = -\frac{2}{3} < 0$, hence $A''(150) < 0$, so by the 2^{nd} derivative test $A(x)$ has a local maximum at $x = 150$, but there is only one critical point, so this is a <u>global</u> <u>maximum</u>.

When $x = 150$, the constraint gives $y = 100 - \frac{1}{3}\cdot 150 = 50$

<u>Answer</u> $\boxed{\text{For maximum area, use } x = 150,\ y = 50}$

5. The questions on this page are about the function $f(x) = x^3 - 9x^2 + 24x - 1$.

(a) Find the intervals on which $f(x)$ increases and on which it decreases.

$f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x-4)(x-2)$

0 2 4

$+ + + + + + + + \mid - - - - - \mid + +$ $f'(x) = 3(x-4)(x-2)$

$f(x)$ increases on $(-\infty, 2) \cup (4, \infty)$
$f(x)$ decreases on $(2, 4)$

(b) Find and identify the local extrema. (Their x values will suffice.)

By 1^{st} derivative test:

Local max at $x = 2$
Local min at $x = 4$

(c) Find the intervals on which $f(x)$ is concave up and on which it is concave down.

$f''(x) = 6x - 18 = 6(x-3)$

0 3

$- - - - - - \mid + + + + + + + +$ $f''(x)$

Concave up on $(3, \infty)$
Concave down on $(-\infty, 3)$

(d) State the locations of all inflection points of $f(x)$. (Their x values will suffice.)

By part (c) above, $f(x)$ has an

inflection point at $x = 3$

(e) Find and identify the global extrema of $f(x)$ on the interval $(3, 6)$.

$f(x)$ has only one critical point $(x = 4)$ in this interval, and by (b) above $f(x)$ has a local minimum there, $\therefore$ a global min.

$f(x)$ has a global minimum at $x = 4$
There is no global maximum