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TEST 3 ♢

MATH 200
April 10, 2026

1. (10 points) Evaluate the limits.

(a) $\lim_{x\to 0} \frac{e^x - e^{-x}}{\cos(x) - x - 1} = \lim_{x\to 0} \frac{e^x + e^{-x}}{-\sin(x) - 1} = \frac{e^0 + e^{-0}}{-\sin(0) - 1} = \frac{1+1}{0-1} = \boxed{-2}$

(form $\frac{e^0 - e^{-0}}{\cos(0) - 0 - 1} = \frac{1-1}{1-1} = \frac{0}{0}$)

(b) $\lim_{x\to 0} x \ln|x| = \lim_{x\to 0} \frac{\ln|x|}{1/x} = \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0} -\frac{1}{x}\frac{x^2}{1} = \lim_{x\to 0} -x$

$= \boxed{0}$

(form $0 \cdot \infty$) (form $\frac{\infty}{\infty}$)

2. (5 points) Suppose $f(x) = \frac{2}{3}x^{3/2} = \frac{2}{3}\sqrt{x}^3$

(a) $f(9) = \frac{2}{3}\sqrt{9}^3 = \frac{2}{3} \cdot 3^3 = 2 \cdot 3^2 = \boxed{18}$

(b) $f'(x) = \frac{2}{3}\frac{3}{2}x^{3/2 - 1} = x^{1/2} = \boxed{\sqrt{x}}$

(c) $f'(9) = \sqrt{9} = \boxed{3}$

(d) Find the linear approximation $L(x)$ of $f(x)$ at the point $x = 9$.

$L(x) = f(9) + f'(9)(x-9)$

$\boxed{L(x) = 18 + 3(x-9)} = \boxed{3x - 9}$

(e) Use your answer from part (d) to find an approximation of $f(9.2)$.

$f(9.2) \approx L(9.2) = 18 + 3(9.2 - 9)$

$= 18 + 3(0.2) = 18 + 0.6 = \boxed{18.6}$

3. (5 points) Suppose f is a function that is continuous on $[1, 3]$ and differentiable on$(1, 3)$. Is it possible for f to satisfy all three of the following properties?

- $f(1) = -1$
- $f(3) = 4$
- $f'(x) \leq 2$ for all values of x.

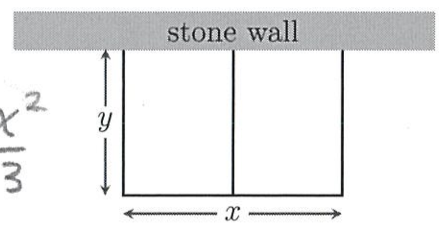
State Yes or No. Use the mean value theorem (and complete sentences) to justify your answer.

NO The mean value theorem asserts that there is a number c in $(1,3)$ for which

$$f'(c) = \frac{f(3)-f(1)}{3-1} = \frac{4-(-1)}{2} = \frac{5}{2} = 2.5$$

But then $f'(c) = 2.5 > 2$, which violates the third condition $f'(c) \leq 2$. So no such f can exist.

4. (5 points) You have a 300 feet of chain link fence to enclose two rectangular pens formed along a stone wall, as illustrated. No fencing is needed along the stone wall. What dimensions (i.e. x feet by y feet) yield the greatest total enclosed area?



Need to maximize

$$\text{area} = xy = x\left(100 - \frac{x}{3}\right) = 100x - \frac{x^2}{3}$$

Constraint:

$$3y + x = 300$$
$$3y = 300 - x$$
$$\boxed{y = 100 - \frac{x}{3}}$$

So we need to find the x that gives a global maximum of

$$A(x) = 100x - \frac{x^2}{3} \text{ on } (0, 300)$$

$$A'(x) = 100 - \frac{2x}{3} = 0$$
$$100 = \frac{2x}{3}$$
$$300 = 2x$$
$$x = 150 \leftarrow \text{critical point}$$

Because $A''(x) = -2/3$ we have $A''(150) < 0$ so there is a global maximum of area at $x = 150$

ANSWER: Use $\boxed{x = 150}$ and $\boxed{y = 100 - \frac{150}{3} = 50}$

5. (25 points) The questions on this page are about the function $f(x) = x^3 - 9x^2 + 24x - 1$.

(a) Find the critical points of $f(x)$.

$$f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x-2)(x-4) = 0$$

Critical Points: $\boxed{x = 2 \quad x = 4}$

(b) Find the intervals on which $f(x)$ increases and on which it decreases.

2   4

$+ + + + \mid - - \mid + + + +$ $f'(x) = 3(x-2)(x-4)$

$f(x)$ increases on $(-\infty, 2) \cup (4, \infty)$

$f(x)$ decreases on $(2, 4)$

(c) Find and identify the local extrema. (Their x values will suffice.)

By first derivative test:

f has a local max at $x = 2$

f has a local min at $x = 4$

(d) Find the intervals on which $f(x)$ is concave up and on which it is concave down.

$$f''(x) = 6x - 18 = 0$$
$$6x = 18$$
$$x = 3$$

3

$- - - - - - - - \ 0 \ + + + + +$ $f''(x)$

f is concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$

(e) State the locations of all inflection points of $f(x)$. (Their x values will suffice.)

$\boxed{x = 3}$ because concavity changes there.