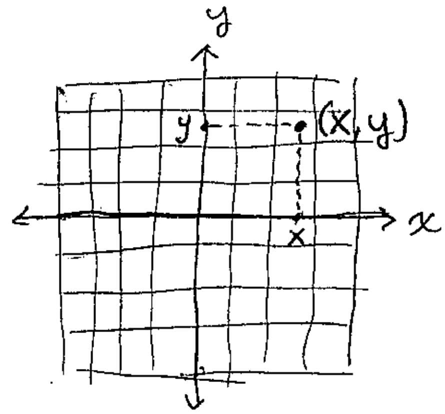
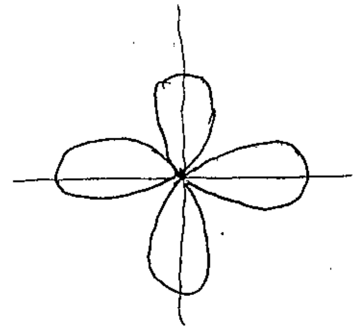
§ 12.2 Polar Coordinates

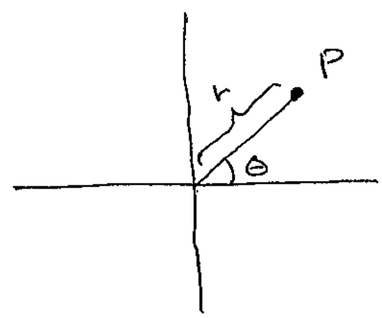
So far, in doing calculus, we've been using the Cartesian coordinate system exclusively.



But some graphs, (think spirograph patterns) have symmetries that are better suited for a different coordinate system - polar coordinates

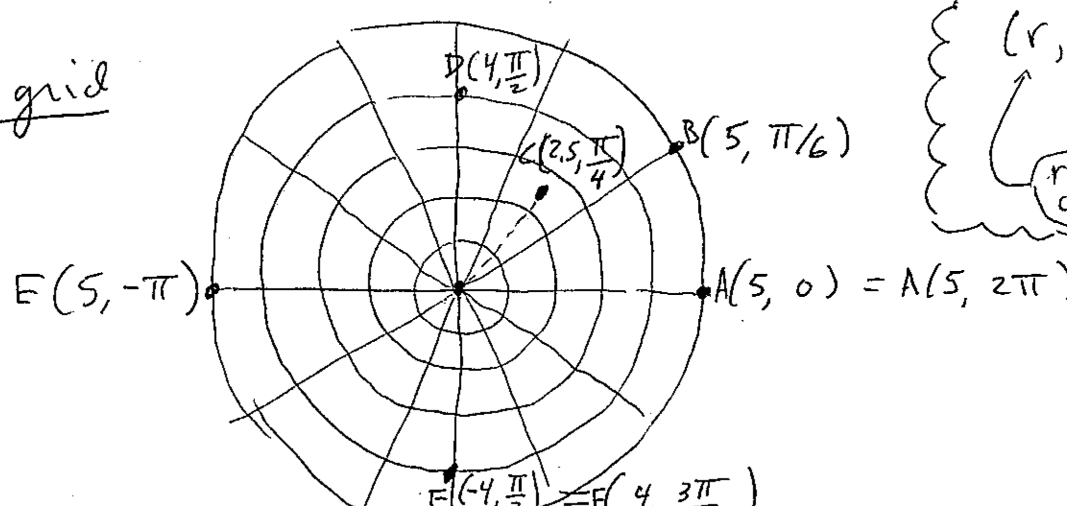


In the polar coordinate system any point on the plane is described by a pair (r, θ)



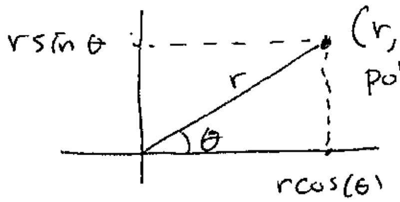
where r is its distance from the origin and θ is its angle of inclination

Polar grid

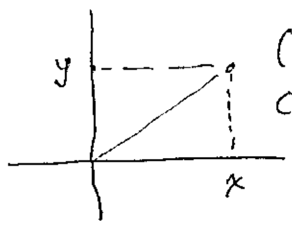


(r, θ)
angular coordinate
radial coordinate

Converting Between Polar and Cartesian



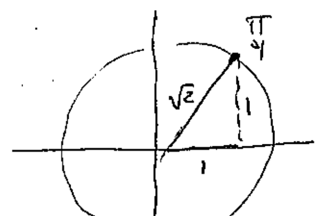
(r, θ) polar $=$ $(r\cos\theta, r\sin\theta)$ cartesian



(x, y) cartesian $=$ (r, θ) polar where $\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan\theta = \frac{y}{x} \end{cases}$

(for a given (x, y), there will be many θ values!)

<u>Ex</u> $\left(\sqrt{2}, \frac{\pi}{4}\right)$ (polar) $= \left(\sqrt{2}\cos\left(\frac{\pi}{4}\right), \sqrt{2}\sin\left(\frac{\pi}{4}\right)\right)$ (cartesian) $= \left(\sqrt{2}\,\frac{\sqrt{2}}{2}, \sqrt{2}\,\frac{\sqrt{2}}{2}\right) = (1, 1)$

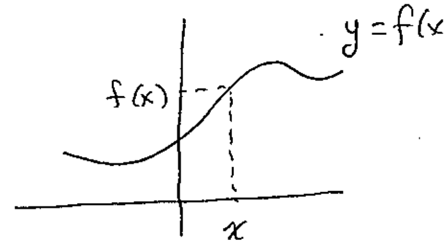


<u>Ex</u> $(1, 1)$ (cartesian) $= \left(\sqrt{1^2 + 1^2}, \theta\right) = \left(\sqrt{2}, \frac{\pi}{4}\right)$ or $\left(\sqrt{2}, \frac{\pi}{4} + k\pi\right)$ for $k = 0, \pm 1, \pm 3 \cdots$

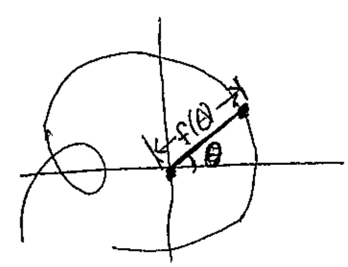
($\tan\theta = \frac{1}{1} = 1$)

<u>Functions</u>

A <u>function</u> $y = f(x)$ can be graphed on a cartesian coordinate system

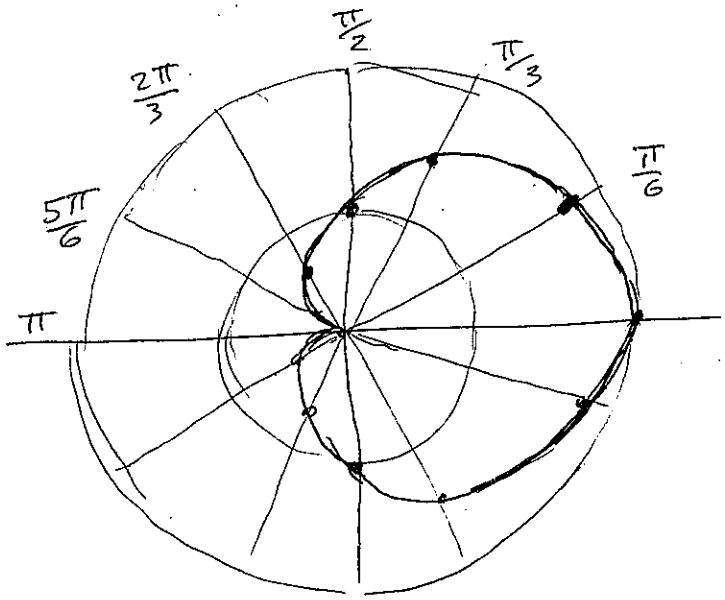


A <u>function</u> $r = f(\theta)$ can be graphed on the polar coordinate system



Ex Graph $r = 1 + \cos\theta$ in polar

θ	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$	$\pm\frac{2\pi}{3}$	$\pm\frac{5\pi}{6}$	$\pm\pi$
$1+\cos\theta$	2	$1+\frac{\sqrt{3}}{2}$ ≈ 1.85	$\frac{3}{2}$	1	$\frac{1}{2}$	$1-\frac{\sqrt{3}}{2}$ ≈ 0.15	0

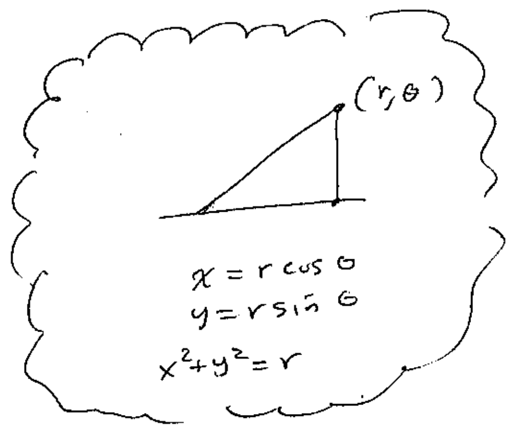


Question: what is the equation for this in the Cartesian plane?

$$r = 1 + \cos\theta$$

$$r^2 = r + r\cos\theta$$

Answer: $x^2 + y^2 = \sqrt{x^2+y^2} + x$



Note that polar expression is simpler! This is why polar coordinates are sometimes useful:

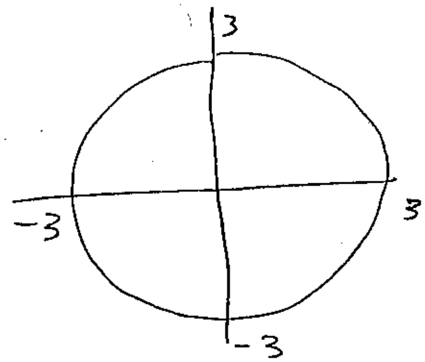
Simpler = better

Examples

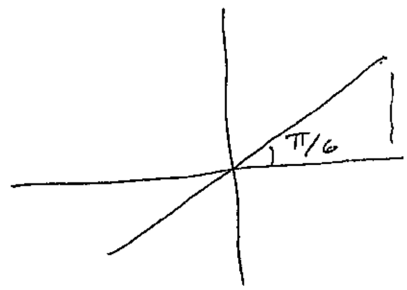
Circle of radius 3

Polar $r = 3$

Cartesian $x^2 + y^2 = 9$



Line, slope $m = \frac{1}{\sqrt{3}}$



Polar $\theta = \frac{\pi}{6}$

Cartesian $y = \frac{1}{2} x$