Chapter 6 Applications of the Definite Integral.

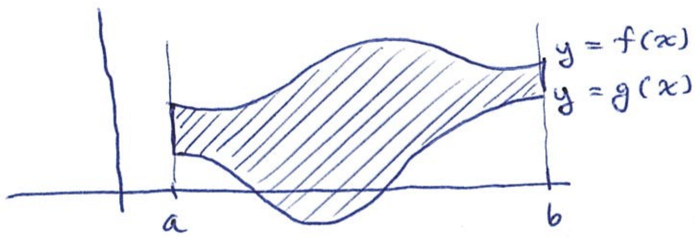
Recall: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x$ (definition)

$\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$ (F.T.C)

The definite integral is a limit, and the F.T.C gives the limit's value when we can find an antiderivative $F(x)$ of the integrand. In this chapter we'll look at some applications of the definite integral. ~~As you~~ Take note that the limit definition is what gives the definite integral its meaning - it should come as no surprise that we'll be using it a lot. Lets start off looking at area again.

Section 6.2 Area Between Two Curves.

Basic Problem Suppose $f(x) \geq g(x)$ on $[a,b]$. What is the area of the shaded region?

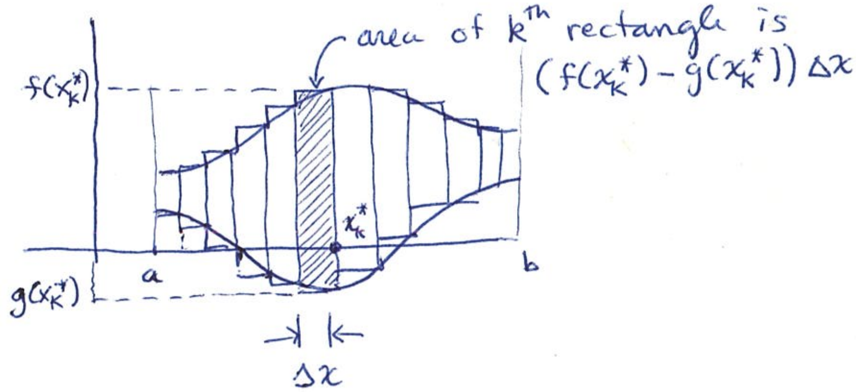


$$A \approx \sum_{k=1}^{n} \left(\begin{array}{c}\text{area of}\\ \text{rectangle}\\ \# k\end{array}\right)$$

$$A \approx \sum_{k=1}^{n} \left(f(x_k^*) - g(x_k^*)\right)\Delta x$$

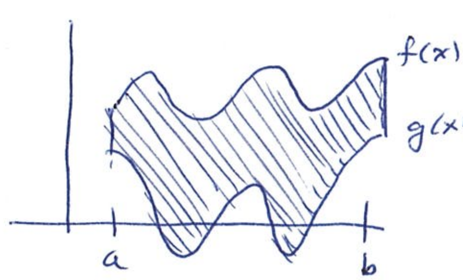
$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(f(x_k^*) - g(x_k^*)\right)\Delta x$$

$$= \int_a^b \left(f(x) - g(x)\right)dx$$



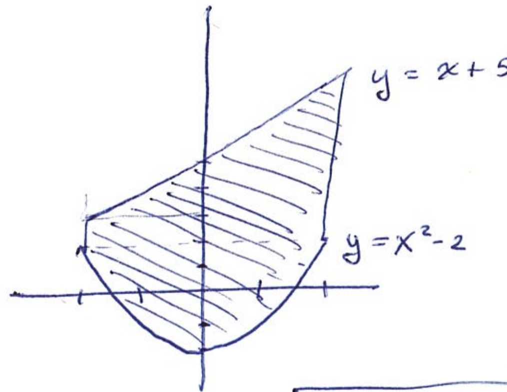
Conclusion: If $f(x) \geq g(x)$ on $[a,b]$ then the shaded area is

$\int_a^b (f(x) - g(x))\,dx$ square units.



Ex

$$A = \int_{-2}^{2} \left(x+5 - (x^2-2)\right) dx = \int_{-2}^{2} (-x^2+x+7)\,dx$$

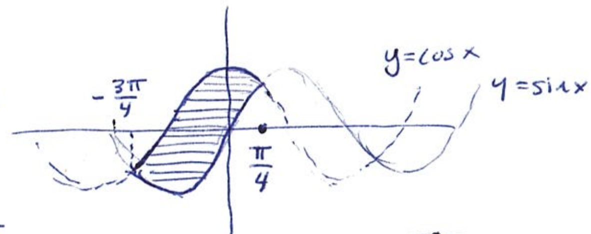


$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 7x\right]_{-2}^{2}$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 7\cdot 2\right) - \left(-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 7(-2)\right)$$

$$= \left(-\frac{8}{3} + 2 + 14\right) - \left(\frac{8}{3} + 2 - 14\right) = -\frac{16}{3} + 28 = \frac{-16}{3} + \frac{84}{3} = \boxed{\frac{68}{3} \text{ sq units}}$$

Ex Find area of shaded region:



$$A = \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left(\cos(x) - \sin(x)\right) dx = \Big[\sin(x) + \cos(x)\Big]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}$$

$$= \left(\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right) - \left(\sin\left(\frac{-3\pi}{4}\right) + \cos\left(\frac{-3\pi}{4}\right)\right)$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} = \boxed{2\sqrt{2} \text{ square units}}$$

Ex Find area between $x = 2y^2 - 4$ and $x = y^2$

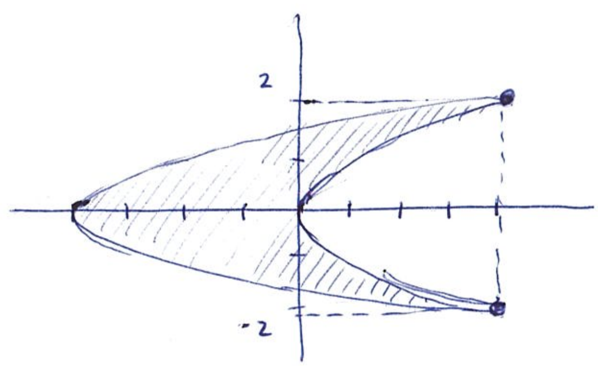
Intersections: $2y^2 - 4 = y^2$

$y^2 - 4 = 0$

$(y+2)(y-2) = 0$

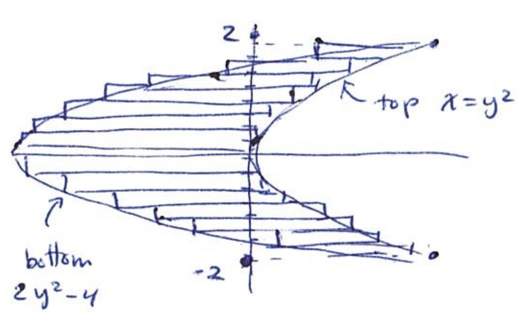
$y = -2 \quad x = 4$

$y = 2 \quad x = 4$



$$A \approx \sum_{k=1}^{n} \left((y_k^*)^2 - \left(2(y_k^*)^2 - 4\right)\right) \Delta y$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left((y_k^*)^2 - \left(2(y_k^*)^2 - 4\right)\right) \Delta y$$



$$= \int_{-2}^{2} \left(y^2 - (2y^2-4)\right) dy$$

$$= \int_{-2}^{2} (-y^2+4)\,dy = \left[-\frac{y^3}{3} + 4y\right]_{-2}^{2}$$

$$= \left(-\frac{2^3}{3} + 4\cdot 2\right) - \left(-\frac{(-2)^3}{3} + 4(-2)\right) = -\frac{16}{3} + 16 = -\frac{16}{3} + \frac{48}{3} = \boxed{\frac{32}{3} \text{ square units.}}$$