Name: Richard

QUIZ 11

MATH 201
February 27, 2025

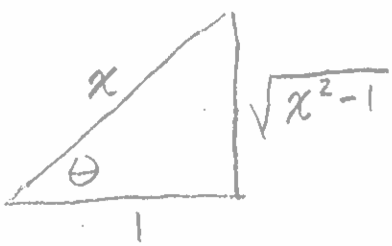
1. $\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \frac{\sqrt{\sec^2(\theta)-1}}{\sec(\theta)} \sec(\theta)\tan(\theta)\,d\theta$

$x = \sec(\theta)$

$dx = \sec(\theta)\tan(\theta)\,d\theta$

$\boxed{\theta = \sec^{-1}(x)}$

$\sec(\theta) = x = \frac{\text{HYP}}{\text{ADJ}}$



$= \int \sqrt{\tan^2(\theta)}\,\tan(\theta)$

$= \int \tan^2(\theta)\,d\theta$

$= \tan(\theta) - \theta + C$

$= \frac{\text{OPP}}{\text{ADJ}} - \sec^{-1}(x) + C$

$= \boxed{\sqrt{x^2-1} - \sec^{-1}(x) + C}$

Check $\frac{d}{dx}\left[\sqrt{x^2-1} - \sec^{-1}(x) + C\right]$

$= \frac{2x}{2\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} + 0 = \frac{x^2-1}{x\sqrt{x^2-1}}$

$= \frac{(x^2-1)\sqrt{x^2-1}}{x\sqrt{x^2-1}\sqrt{x^2-1}} = \frac{\cancel{(x^2-1)}\sqrt{x^2-1}}{x\cancel{(x^2-1)}} = \frac{\sqrt{x^2-1}}{x}$ YES

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1. $\int_0^1 \sqrt{1-x^2}\,dx = \int_{\sin^{-1}(0)}^{\sin^{-1}(1)} \sqrt{1-\sin^2(\theta)}\cos(\theta)\,d\theta$

$x = \sin(\theta) \implies \theta = \sin^{-1}(x)$

$dx = \cos(\theta)\,d\theta$

$= \int_0^{\pi/2} \sqrt{\cos^2(\theta)}\cos(\theta)\,d\theta$

$= \int_0^{\frac{\pi}{2}} \cos^2(\theta)\,d\theta$

$= \left[\frac{1}{2}\left(\theta + \cos(\theta)\sin(\theta)\right)\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left(\frac{\pi}{2} + \cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)\right) - \frac{1}{2}\left(0 + \cos(0)\sin(0)\right)$

$= \frac{1}{2}\left(\frac{\pi}{2} + (0)(1)\right) - \frac{1}{2}\left(0 + 1\cdot 0\right)$

$= \boxed{\frac{\pi}{4}}$