Name: Richard

QUIZ 12

MATH 201
March 4, 2025

1. $\int \frac{81}{x^3 - 9x^2}\,dx =$ $\int \frac{81}{x^2(x-9)}\,dx = \int \frac{-1}{x} - \frac{9}{x^2} + \frac{1}{x-9}\,dx$

$= -\ln|x| + \frac{9}{x} + \ln|x-9| + C$

$= \boxed{\frac{9}{x} + \ln\left|\frac{x-9}{x}\right| + C}$

$\frac{81}{x^2(x-9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-9}$

$\boxed{81 = Ax(x-9) + B(x-9) + Cx^2}\ (*)$

Put $x = 9$

$81 = A\,9(9-9) + B(9-9) + C\cdot 9^2$

$81 = 81C$

$\boxed{C = 1}$

Put $x = 0$:

$81 = A\,0(0-9) + B(0-9) + C\cdot 0^2$

$81 = -9B$

$\boxed{B = -9}$

Now Equation $(*)$ is $81 = Ax(x-9) - 9(x-9) + x^2$

Put $x = 10$:

$81 = A\cdot 10(10-9) - 9(10-9) + 10^2$

$81 = 10A - 9 + 100$

$-10 = 10A$

$\boxed{A = -1}$

Check:

$\frac{d}{dx}\left[\frac{9}{x} + \ln\left|\frac{x-9}{x}\right| + C\right]$

$= \frac{-9}{x^2} + \frac{\frac{1\cdot x - (x-9)\cdot 1}{x^2}}{\frac{x-9}{x}}$

$= \frac{-9}{x^2} + \frac{x}{x-9}\cdot\frac{(x-x+9)}{x^2}$

$= -\frac{9}{x^2} + \frac{9x}{(x-9)x^2}$ = YES

$= \frac{-9(x-9)}{x^2(x-9)} + \frac{9x}{x^2(x-9)}$

$= \frac{-9x + 81 + 9x}{x^2(x-9)} = \frac{81}{x^3 - 9x^2}$

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1. $\int \frac{x}{(x+3)^2}\,dx = \int \frac{1}{x+3} - \frac{3}{(x+3)^2}\,dx = \boxed{\ln|x+3| + \frac{3}{x+3} + C}$

$$\frac{x}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

$$x = A(x+3) + B$$

$$x = Ax + 3A + B$$

$$\Downarrow$$

$$1 \cdot x + 0 = Ax + (3A + B)$$

$$\Downarrow$$

$$\boxed{A = 1}$$

$$3A + B = 0$$

$$3 \cdot 1 + B = 0$$

$$\boxed{B = -3}$$

Check:

$$\frac{d}{dx}\left[\ln|x+3| + \frac{3}{x+3} + C\right]$$

$$= \frac{1}{x+3} - \frac{3}{(x+3)^2} + 0$$

$$= \frac{x+3}{(x+3)^2} - \frac{3}{(x+3)^2}$$

$$= \frac{x}{(x+3)^2}$$

✓ YES