Name: Richard

Quiz 13

MATH 201
March 6, 2025

1. $\displaystyle\int \frac{6e^x}{e^{2x}+2e^x-8}\,dx = \int \frac{6e^x}{(e^x)^2+2e^x-8}\,dx = \int \frac{6}{u^2+2u-8}\,du$

$u = e^x$
$du = e^x\,dx$

$$= \int \frac{6}{(u+4)(u-2)}\,du$$

$$\frac{6}{(u+4)(u-2)} = \frac{A}{u+4} + \frac{B}{u-2}$$

$$6 = A(u-2) + B(u+4)$$

$$6 = Au - 2A + Bu + 4B$$

$$0u + 6 = (A+B)u - 2A + 4B$$

$$\Downarrow$$

$$\begin{cases} A + B = 0 \\ -2A + 4B = 6 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} A + B = 0 \\ -A + 2B = 3 \end{cases}$$

$$3B = 3$$

$$\boxed{B = 1}$$

and $A + B = 0 \Rightarrow \boxed{A = -1}$

$$= \int \frac{A}{u+4} + \frac{B}{u-2}\,du$$

$$= \int \frac{-1}{u+4} + \frac{1}{u-2}\,du$$

$$= -\ln|u+4| + \ln|u-2| + C$$

$$= \ln\left|\frac{u-2}{u+4}\right| + C$$

$$= \boxed{\ln\left|\frac{e^x-2}{e^x+4}\right| + C}$$

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1. $\displaystyle\int \frac{\cos(x)}{\sin^2(x)+\sin(x)}\,dx = \int \frac{1}{u^2+u}\,du = \int \frac{1}{u(u+1)}\,du$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int \frac{A}{u} + \frac{B}{u+1}\,du$$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$1 = A(u+1) + Bu$$

$$1 = Au + A + Bu$$

$$0u + 1 = (A+B)u + A$$

$$\Downarrow$$

$$\boxed{A = 1}$$

$$A + B = 0 \implies \boxed{B = -1}$$

$$= \int \frac{1}{u} - \frac{1}{u+1}\,du$$

$$= \ln|u| - \ln|u+1| + C$$

$$= \ln\left|\frac{u}{u+1}\right| + C$$

$$= \boxed{\ln\left|\frac{\sin(x)}{\sin(x)+1}\right| + C}$$