Name: Richard

QUIZ 14

MATH 201
March 18, 2025

1. $\int_2^\infty \frac{1}{x^2 - x}\,dx =$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x^2 - x}\,dx$$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x(x-1)}\,dx$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$1 = A(x-1) + Bx$$

$$\boxed{0x + 1 = (A+B)x - A}$$

$$\begin{cases} -A = 1 \\ A + B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} A = -1 \\ -1 + B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} A = -1 \\ B = 1 \end{cases}$$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x-1} - \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \Big[\ln|x-1| - \ln|x|\Big]_2^b$$

$$= \lim_{b\to\infty} \left[\ln\left|\frac{x-1}{x}\right|\right]_2^b$$

$$= \lim_{b\to\infty} \left(\ln\left|\frac{b-1}{b}\right| - \ln\left|\frac{2-1}{2}\right|\right)$$

$$= \ln\left|\lim_{b\to\infty} \frac{b-1}{b}\right| - \ln\left|\frac{1}{2}\right|$$

$$= \ln|1| + \ln\left(\left(\frac{1}{2}\right)^{-1}\right) = 0 + \ln(2) = \boxed{\ln(2)}$$

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1. $\displaystyle\int_1^\infty \frac{(\tan^{-1}(x))^2}{x^2+1}\,dx =$

$$u = \tan^{-1}(x)$$
$$du = \frac{1}{x^2+1}\,dx$$

$$= \lim_{b\to\infty} \int_1^b (\tan^{-1}(x))^2 \frac{1}{x^2+1}\,dx$$

$$= \lim_{b\to\infty} \int_{\tan^{-1}(1)}^{\tan^{-1}(b)} u^2\,du$$

$$= \lim_{b\to\infty} \left[\frac{u^3}{3}\right]_{\frac{\pi}{4}}^{\tan^{-1}(b)}$$

$$= \lim_{b\to\infty} \left(\frac{(\tan^{-1}(b))^3}{3} - \frac{\left(\pi/4\right)^3}{3}\right)$$

$$= \frac{\left(\frac{\pi}{2}\right)^3}{3} - \frac{\left(\frac{\pi}{4}\right)^3}{3} = \frac{1}{3}\left(\frac{\pi^3}{8} - \frac{\pi^3}{64}\right)$$

$$\begin{array}{r} \overset{1}{6}4 \\ \cdot\, 3 \\ \hline 192 \end{array}$$

$$= \frac{\pi^3}{3}\left(\frac{8}{64} - \frac{1}{64}\right) = \boxed{\frac{7\pi^3}{192}}$$