Name: Richard

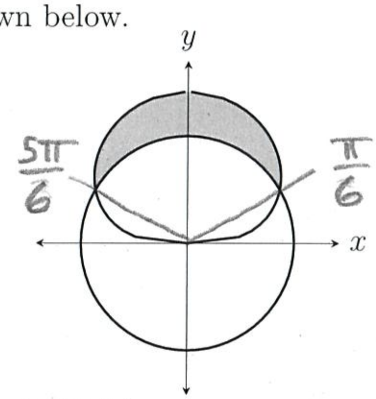
QUIZ 16

MATH 201

April 1, 2025

1. The graphs of the polar equations $r = \sqrt{\sin(\theta)}$ and $r = \frac{1}{\sqrt{2}}$ are shown below.

Find the area of the shaded region.



To find the intersections, solve:

$$\sqrt{\sin(\theta)} = \frac{1}{\sqrt{2}}$$

$$\sqrt{\sin(\theta)}^2 = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\sin(\theta) = \frac{1}{2} \Rightarrow \boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\left(\sqrt{\sin(\theta)}^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right) d\theta$$

$$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(\sin(\theta) - \frac{1}{2}\right) d\theta$$

$$= \frac{1}{2}\left[-\cos\theta - \frac{\theta}{2}\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{1}{2}\left(\left(-\cos\left(\frac{5\pi}{6}\right) - \frac{5\pi}{12}\right) - \left(-\cos\left(\frac{\pi}{6}\right) - \frac{\pi}{12}\right)\right)$$

$$= \frac{1}{2}\left(-\left(-\frac{\sqrt{3}}{2}\right) - \frac{5\pi}{12} + \frac{\sqrt{3}}{2} + \frac{\pi}{12}\right)$$

$$= \frac{1}{2}\left(\sqrt{3} - \frac{4\pi}{12}\right) = \boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{6} \text{ square units}}$$

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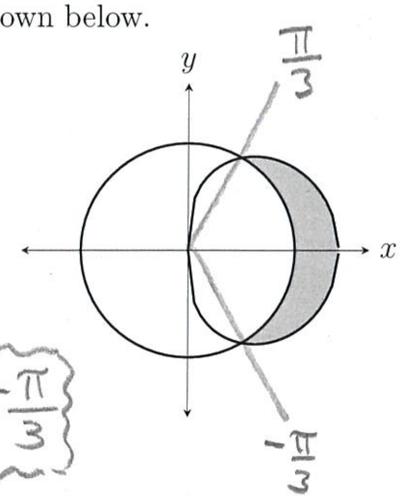
Quiz 16

MATH 201
April 1, 2025

1. The graphs of the polar equations $r = \sqrt{2\cos(\theta)}$ and $r = 1$ are shown below. Find the area of the shaded region.



To find the intersection points, solve

$$\sqrt{2\cos(\theta)} = 1$$

$$\sqrt{2\cos(\theta)}^{\,2} = 1^2$$

$$2\cos(\theta) = 1$$

$$\cos(\theta) = 1/2 \Rightarrow \boxed{\theta = \frac{\pi}{3}, \frac{-\pi}{3}}$$

$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}\left(\sqrt{2\cos(\theta)}^{\,2} - 1^2\right) d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (2\cos\theta - 1)\, d\theta = \frac{1}{2}\Big[2\sin(\theta) - \theta\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left(\left(2\sin\left(\frac{\pi}{3}\right) - \frac{\pi}{3}\right) - \left(2\sin\left(\frac{-\pi}{3}\right) - \left(\frac{-\pi}{3}\right)\right)\right)$$

$$= \frac{1}{2}\left(2\cdot\frac{\sqrt{3}}{2} - \frac{\pi}{3} - \left(-\frac{2\sqrt{3}}{2} + \frac{\pi}{3}\right)\right)$$

$$= \frac{1}{2}\left(2\sqrt{3} - \frac{2\pi}{3}\right) = \boxed{\sqrt{3} - \frac{\pi}{3} \text{ square units}}$$