Name: Richard

Quiz 17

MATH 201
April 3, 2025

1. Does the sequence $\left\{ n \sin\left(\frac{3}{n}\right) \right\}_{n=1}^{\infty}$ converge or diverge? If it converges, find the limit.

$$\lim_{n\to\infty} n \sin\left(\frac{3}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{3}{n}\right)}{\frac{1}{n}} = \lim_{n\to 0} \frac{\cos\left(\frac{3}{n}\right)\left(\frac{-3}{n^2}\right)}{-\frac{1}{n^2}}$$

form $\infty \cdot 0$ form $\frac{0}{0}$ Apply L'Hopital

$$= \lim_{n\to\infty} 3\cos\left(\frac{3}{n}\right) = 3\cos(0) = 3 \cdot 1 = \boxed{3}$$

$\boxed{\text{The sequence converges to } \boxed{3}}$

2. Does the series $\sum_{k=0}^{\infty} \frac{2}{\pi^k}$ converge or diverge? If it converges, find the sum.

$$\sum_{k=0}^{\infty} \frac{2}{\pi^k} = 2 + \frac{2}{\pi} + \frac{2}{\pi^2} + \frac{2}{\pi^3} + \frac{2}{\pi^4} + \cdots$$

geometric series, $a = 2$, $r = \frac{1}{\pi} < 1$.

<u>Converges</u> $\displaystyle\sum_{k=0}^{\infty} \frac{2}{\pi^k} = \frac{a}{1-r} = \frac{2}{1-\frac{1}{\pi}}$

$$= \frac{2}{\frac{\pi - 1}{\pi}} = \boxed{\frac{2\pi}{\pi - 1}}$$

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1. Does the sequence $\left\{ \dfrac{4n + 7e^n}{n - 3e^n} \right\}_{n=1}^{\infty}$ converge or diverge? If it converges, find the limit.

$$\lim_{n\to\infty} \underbrace{\frac{4n+7e^n}{n-3e^n}}_{\text{form } \frac{\infty}{\infty}} \underset{\text{Apply L'Hôptial}}{=} \lim_{n\to\infty} \underbrace{\frac{4+7e^n}{1-3e^n}}_{\text{form } \frac{\infty}{\infty}} \underset{\text{Apply L'Hôpital}}{=} \lim_{n\to\infty} \frac{7e^n}{-3e^n} = \boxed{-\frac{7}{3}}$$

Sequence converges to $\boxed{-7/3}$

2. Does the series $\displaystyle\sum_{k=0}^{\infty} \frac{5}{(-6)^k}$ converge or diverge? If it converges, find the *sum*.

$$\sum_{k=0}^{\infty} \frac{5}{(-6)^k} = 5 - \frac{5}{6} + \frac{5}{36} - \cdots$$

geometric series, $a = 5$ $r = -\frac{1}{6}$, and $|r| < 1$. Therefore it converges.

$$\sum_{k=0}^{\infty} \frac{5}{(-6)^k} = \frac{5}{1-(-\frac{1}{6})} = \frac{5}{1+\frac{1}{6}} = \frac{5}{\frac{6+1}{6}} = \boxed{\frac{30}{7}}$$