Name: Richard

Quiz 18

MATH 201

April 8, 2025

Determine whether each series converges or diverges. If it converges, state the sum if possible. Explain.

1. $\displaystyle\sum_{k=0}^{\infty} \frac{2k}{(k^2+3)^9}$

Let $f(x) = \dfrac{2x}{(x^2+3)^9}$ so this series is $\displaystyle\sum_{k=1}^{\infty} f(k)$.

Note that $f(k) > 0$ and $f'(k) = \dfrac{2(k^2+3)^9 - 18k(k^2+3)^8 2k}{(k^2+3)^{18}}$

$= \dfrac{2(k^2+3)^8(k^2+3-18k^2)}{(k^2+3)^{18}} = \dfrac{2(k^2+3)(3-17k^2)}{(k^3+3)^{18}} < 0$. Thus the series terms are positive and decrease so the integral test applies:

$$\int_0^{\infty} \frac{2x}{(x^2+3)^9}\,dx = \lim_{b\to\infty} \int_0^b (x^2+3)^{-9}\, 2x\,dx$$

$u = x^2+3$

$du = dx$

$$= \lim_{b\to\infty} \int_{0^2+3}^{b^2+3} u^{-9}\,du = \lim_{b\to\infty} \left[-\frac{1}{8u^8}\right]_3^{b^2+3} = \frac{1}{8\cdot 3^8} < \infty$$

The integral converges so the series ~~dive~~ converges also

2. $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{k}{k+9}}$

$$\lim_{k\to\infty} \sqrt{\frac{k}{k+9}} = \sqrt{\lim_{k\to\infty} \frac{k}{k+9}}$$

$$= \sqrt{1} = 1 \neq 0.$$

Therefore the series diverges by the divergence test.

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Determine whether each series converges or diverges. If it converges, state the sum if possible. Explain.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{10}{k^2+9}$ Let $f(x) = \dfrac{10}{x^2+9}$ so this series is $\displaystyle\sum_{k=1}^{\infty} f(k)$.

Note that $f(x) > 0$ and $f'(x) = \dfrac{20x}{(x^2+9)} > 0$ so the series terms are positive and the decrease, hence the integral test applies.

$$\int_1^{\infty} \frac{10}{x^2+9}\,dx = \lim_{b\to\infty} \int_1^b \frac{10}{x^2+3^2}\,dx = \lim_{b\to\infty} \left[\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_1^b$$

$$= \lim_{b\to\infty} \left(\frac{10}{3}\tan^{-1}\left(\frac{b}{3}\right) - \frac{10}{3}\tan^{-1}\left(\frac{1}{3}\right)\right) = \frac{10}{3}\frac{\pi}{2} - \frac{10}{3}\tan^{-1}\left(\frac{1}{3}\right)$$

$< \infty$, so the integral converges so the series $\boxed{\displaystyle\sum_{k=1}^{\infty} \frac{10}{k^2+9} \text{ converges too.}}$

2. $$\sum_{k=0}^{\infty} \frac{2^k+3^k}{4^k} = \sum_{k=0}^{\infty}\left(\frac{2^k}{4^k} + \frac{3^k}{4^k}\right) = \sum_{k=0}^{\infty}\left(\left(\frac{2}{4}\right)^k + \left(\frac{3}{4}\right)^k\right)$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k + \sum_{k=0}^{\infty}\left(\frac{3}{4}\right)^k$$ ← both are convergent geometric series!

$$= \frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{3}{4}} = \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{4}}$$

$$= 2 + 4 = \boxed{6}$$ ← series converges to 6