

Use substitution to find the following integrals. State clearly what your substitution is. Show all steps.

$$1. \int \frac{\sin(x)}{\sqrt{\cos(x)}} dx = \int (\cos(x))^{-1/2} \sin(x) dx$$

$$= \int u^{-1/2} (-1) du = - \int u^{-1/2} du$$

$$= - \frac{u^{-1/2+1}}{-1/2+1} + C$$

$$= - \frac{u^{1/2}}{1/2} + C = -2\sqrt{u} + C$$

$$= \boxed{-2\sqrt{\cos(x)} + C}$$

$$\begin{cases} u = \cos(x) \\ \frac{du}{dx} = -\sin(x) \\ du = -\sin(x) dx \\ (-1) du = \sin(x) dx \end{cases}$$

$$2. \int_0^{\ln(2)} \frac{e^x}{5+e^x} dx = \int_6^{5+e^{\ln(2)}} \frac{1}{u} e^x dx = \int_{5+e^0}^{5+e^{\ln(2)}} \frac{1}{u} du$$

$$= \int_6^7 \frac{1}{u} du = [\ln|u|]_6^7$$

$$= \ln|7| - \ln|6|$$

$$= \boxed{\ln\left|\frac{7}{6}\right|}$$

$$\begin{cases} u = 5+e^x \\ \frac{du}{dx} = e^x \\ du = e^x dx \end{cases}$$

Use substitution to find the following integrals. State clearly what your substitution is. Show all steps.

$$1. \int \frac{(3+2\ln|x|)^7}{x} dx = \int (3+2\ln|x|)^7 \frac{1}{x} dx = \int u^7 \frac{1}{2} du$$

$$\begin{aligned} u &= 3 + 2\ln|x| \\ \frac{du}{dx} &= 2 \cdot \frac{1}{x} \\ du &= \frac{2}{x} dx \\ \frac{1}{2} du &= \frac{1}{x} dx \end{aligned}$$

$$= \frac{1}{2} \int u^7 du = \frac{1}{2} \frac{u^8}{8} + C$$

$$= \boxed{\frac{(3+2\ln|x|)^8}{16} + C}$$

$$2. \int_0^{\ln \sqrt{3}} \frac{e^x}{1+(e^x)^2} dx = \int_0^{\ln \sqrt{3}} \frac{1}{1+(e^x)^2} e^x dx = \int_{e^0}^{e^{\ln \sqrt{3}}} \frac{1}{1+u^2} du$$

$$\begin{aligned} u &= e^x \\ \frac{du}{dx} &= e^x \\ du &= e^x dx \end{aligned}$$

$$= \int_1^{\sqrt{3}} \frac{1}{1+u^2} du = \left[ \tan^{-1}(u) \right]_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \boxed{\frac{\pi}{12}}$$