Name: Richard

QUIZ 20

MATH 201
April 15, 2025

1. Use the ratio or root test to determine whether the series converges:

$$\sum_{k=1}^{\infty} \left(\frac{k}{3k+1}\right)^k$$

Let's try the root test.

$$\lim_{k\to\infty} \sqrt[k]{|a_k|} = \lim_{k\to\infty} \sqrt[k]{\left(\frac{k}{3k+1}\right)^k}$$

$$= \lim_{k\to\infty} \frac{k}{3k+1}$$

$$= \lim_{k\to\infty} \frac{k}{3k+1} \frac{1/k}{1/k}$$

$$= \lim_{k\to\infty} \frac{1}{3+1/k}$$

$$= \frac{1}{3+0} = \frac{1}{3} < 1.$$

Because the limit is less than 1, the root test tells us that the $\boxed{\text{series converges}}$

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1. Use the ratio or root test to determine whether the series converges:

$$\sum_{k=1}^{\infty} \frac{k+1}{2^k}$$

Let's try the ratio test.

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \frac{\frac{(k+1)+1}{2^{k+1}}}{\frac{k+1}{2^k}}$$

$$= \lim_{k\to\infty} \frac{2^k}{k+1} \cdot \frac{k+2}{2^{k+1}}$$

$$= \lim_{k\to\infty} \frac{1}{2} \frac{k+2}{k+1}$$

$$= \lim_{k\to\infty} \frac{1}{2} \frac{k+2}{k+1} \frac{1/k}{1/k}$$

$$= \lim_{k\to\infty} \frac{1}{2} \frac{1+2/k}{1+1/k}$$

$$= \frac{1}{2} \frac{1+0}{1+0} = \frac{1}{2} < 1$$

Because $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} < 1$, the ratio test tells us that the series $\sum_{k=1}^{\infty} \frac{k+1}{2^k}$ $\boxed{\text{converges}}$