Name: Richard

QUIZ 21 ♠

MATH 201
April 18, 2024

1. Use any appropriate test to determine whether the series converges.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}\, e^{\sqrt{k}}}$$

Let $f(x) = \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}$, so this series is $\sum_{k=1}^{\infty} f(k)$.

Notice that $f(x) > 0$ for any $x > 0$ and $f(x)$ is a decreasing function because its denominator increases with x. Therefore the integral test applies.

$$\int_1^{\infty} \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = \int_1^{\infty} e^{-\sqrt{x}} \frac{1}{\sqrt{x}}\, dx = \lim_{b\to\infty} \int_1^b e^{-\sqrt{x}} \frac{1}{\sqrt{x}}\, dx$$

$u = -\sqrt{x}$

$du = \frac{-1}{2\sqrt{x}}\, dx$

$-2\,du = \frac{1}{\sqrt{x}}\, dx$

$$= \lim_{b\to\infty} \int_{-\sqrt{1}}^{-\sqrt{b}} e^u (-2)\, du$$

$$= -2 \lim_{b\to\infty} \int_{-1}^{-\sqrt{b}} e^u\, du$$

$$= -2 \lim_{b\to\infty} \Big[e^u \Big]_{-1}^{-\sqrt{b}} = -2 \lim_{b\to\infty} \left(e^{-\sqrt{b}} - e^{-1} \right)$$

$$= -2 \lim_{b\to\infty} \left(\frac{1}{e^{\sqrt{b}}} - \frac{1}{e} \right) = -2 \left(0 - \frac{1}{e} \right) = \boxed{\frac{2}{e}}$$

Conclusion: Because the integral converges, the $\boxed{\text{series converges}}$ by the integral test.

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QUIZ 21 ◊

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1. Use any appropriate test to determine whether the series converges.

$$\sum_{k=1}^{\infty} \frac{3+\cos(5k)}{k^3}$$

Because $-1 \leq \cos(5k) \leq 1$ we know that

$$2 \leq 3+\cos(5k) \leq 4$$

In particular, this means $\frac{3+\cos(5k)}{k^3}$ is always positive and $\frac{3+\cos(5k)}{k^3} \leq \frac{4}{k^3}$

Consequently $\boxed{\sum_{k=1}^{\infty} \frac{3+\cos(5k)}{k^3} \text{ converges}}$ by comparison with the convergent p-series $\sum_{k=1}^{\infty} \frac{4}{k^3}$.

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QUIZ 21 ♣

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1. Use any appropriate test to determine whether the series converges.
$\sum_{k=1}^{\infty} \frac{k^5}{5^k}$

Ratio Test: $\displaystyle \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{\frac{(k+1)^5}{5^{k+1}}}{\frac{k^5}{5^k}} \right|$

$$= \lim_{k\to\infty} \frac{(k+1)^5}{k^5} \frac{5^k}{5^{k+1}} = \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^5 \frac{5^k}{5^k \cdot 5}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^5 \frac{1}{5} = \frac{1}{5} \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^5$$

$$= \frac{1}{5} \left(\lim_{k\to\infty} \frac{k+1}{k}\right)^5 = \frac{1}{5} \cdot 1^5 = \frac{1}{5} < 1.$$

Because the limit is less than 1, the $\boxed{\text{series converges}}$ by the ratio test.

1. Use any appropriate test to determine whether the series converges.

$$\frac{2}{3}+\frac{3}{8}+\frac{4}{15}+\frac{5}{24}+\frac{6}{35}+\frac{7}{48}+\cdots$$

$$= \sum_{k=2}^{\infty} \frac{k}{k^2-1}$$

Notice that $\frac{k}{k^2-1} > \frac{k}{k^2}$ because increasing the denominator (by dropping the -1) decreases the value of the fraction. Therefore

$$\frac{k}{k^2} < \frac{k}{k^2-1}$$

$$\frac{1}{k} < \frac{k}{k^2-1}$$

Because $\sum_{k=2}^{\infty} \frac{1}{k}$ <u>diverges</u> (harmonic series) it follows that $\sum_{k=2}^{\infty} \frac{k}{k^2-1}$ also <u>diverges</u> by the comparison test.