Name: Richard

QUIZ 21

MATH 201
April 17, 2025

1. Use any appropriate test to determine convergence:

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k^{1/2}+k}}$$

This is an alternating series $\sum (-1)^k a_k$ with

$a_k = \frac{1}{\sqrt{\sqrt{k}+k}}$.

(1) Moreover $a_k = \frac{1}{\sqrt{\sqrt{k}+k}}$ decreases as k increases because the denominator increases.

(2) Further $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{1}{\sqrt{\sqrt{k}+k}} = 0$

(Because numerator is 1 and denominator approaches ∞.)

By the alternating series test, the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{\sqrt{k}+k}}$ $\boxed{\text{converges.}}$

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1. Use any appropriate test to determine convergence:

$$\frac{1}{1!}+\frac{4}{2!}+\frac{9}{3!}+\frac{16}{4!}+\frac{25}{5!}+\cdots = \sum_{k=1}^{\infty}\frac{k^2}{k!}$$

Lets try the <u>ratio</u> <u>test</u>.

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\frac{\frac{(k+1)^2}{(k+1)!}}{\frac{k^2}{k!}}$$

$$= \lim_{k\to\infty}\frac{(k+1)^2}{(k+1)!}\frac{k!}{k^2} = \lim_{k\to\infty}\frac{(k+1)^2}{k^2(k+1)}$$

$$= \lim_{k\to\infty}\frac{k+1}{k^2} = 0 < 1$$

Therefore the series $\sum_{k=1}^{\infty}\frac{k^2}{k!}$

<u>converges</u> by the <u>ratio</u> <u>test</u>.