Name: Richard

QUIZ 22 ♠

MATH 201

April 23, 2024

1. Find the Taylor polynomial $p_5(x)$ for $\sin(x)$ centered at $a = 0$. Show all work.

$f^{(0)}(x) = \sin(x)$ $\quad f^{(0)}(0) = \sin(0) = 0$

$f^{(1)}(x) = \cos(x)$ $\quad f^{(1)}(0) = \cos(0) = 1$

$f^{(2)}(x) = -\sin(x)$ $\quad f^{(2)}(0) = -\sin(0) = 0$

$f^{(3)}(x) = -\cos(x)$ $\quad f^{(3)}(0) = -\cos(0) = -1$

$f^{(4)}(x) = \sin(x)$ $\quad f^{(4)}(0) = \sin(0) = 0$

$f^{(5)}(x) = \cos(x)$ $\quad f^{(5)}(0) = \cos(0) = 1$

$$p_5(x) = \sum_{k=0}^{5} \frac{f^{(k)}(0)}{k!} x^k =$$

$$= \frac{f^{(0)}(0)}{0!}x^0 + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5$$

$$= \frac{0}{1}\cdot 1 + \frac{1}{1}x^1 + \frac{0}{2}x^2 - \frac{1}{6}x^3 + \frac{0}{4!}x^4 + \frac{1}{5!}x^5$$

$$= \boxed{x - \frac{1}{6}x^3 + \frac{1}{120}x^5}$$

Name: Richard

Quiz 22 ♢

MATH 201
April 23, 2024

1. Find the Taylor polynomial $p_4(x)$ for e^{-x} centered at $a = 0$. Show all work.

$f^{(0)}(x) = e^{-x}$ $\quad$ $f^{(0)}(0) = e^{-0} = 1$

$f^{(1)}(x) = -e^{-x}$ $\quad$ $f^{(1)}(0) = -e^{-0} = -1$

$f^{(2)}(x) = e^{-x}$ $\quad$ $f^{(2)}(0) = e^{-0} = 1$

$f^{(3)}(x) = -e^{-x}$ $\quad$ $f^{(3)}(0) = -e^{-0} = -1$

$f^{(4)}(x) = e^{-x}$ $\quad$ $f^{(4)}(0) = e^{-0} = 1$

$$p_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} x^k$$

$$= \frac{f^{(0)}(0)}{0!}x^0 + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= \frac{1}{1}x^0 - \frac{1}{1}x^1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4$$

$$= \boxed{1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}}$$

Name: Richard | QUIZ 21 ♣ | MATH 201
April 23, 2024

1. Find the Taylor polynomial $p_4(x)$ for $\cos(x)$ centered at $a = 0$. Show all work.

$f^{(0)}(x) = \cos(x)$ $\quad$ $f^{(0)}(0) = \cos(0) = 1$

$f^{(1)}(x) = -\sin(x)$ $\quad$ $f^{(1)}(0) = -\sin(0) = 0$

$f^{(2)}(x) = -\cos(x)$ $\quad$ $f^{(2)}(0) = -\cos(0) = -1$

$f^{(3)}(x) = \sin(x)$ $\quad$ $f^{(3)}(0) = \sin(0) = 0$

$f^{(4)}(x) = \cos(x)$ $\quad$ $f^{(4)}(0) = \cos(0) = 1$

$$p_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!} x^k$$

$$= \frac{f^{(0)}(0)}{0!}x^0 + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{3}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= \frac{1}{1}\cdot 1 + \frac{0}{1!}x - \frac{1}{2}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4$$

$$= \boxed{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4}$$

Name: Richard

QUIZ 22 ♡

MATH 201
April 23, 2024

1. Find the Taylor polynomial $p_4(x)$ for $\ln(x)$ centered at $a = 1$. Show all work.

$f^{(0)}(x) = \ln(x)$ $\quad$ $f^{(0)}(1) = \ln(1) = 0$

$f^{(1)}(x) = \frac{1}{x}$ $\quad$ $f^{(1)}(1) = \frac{1}{1} = 1$

$f^{(2)}(x) = -\frac{1}{x^2}$ $\quad$ $f^{(2)}(1) = \frac{-1}{1^2} = -1$

$f^{(3)}(x) = \frac{2}{x^3}$ $\quad$ $f^{(3)}(1) = \frac{2}{1^3} = 2$

$f^{(4)}(x) = \frac{-6}{x^4}$ $\quad$ $f^{(4)}(1) = \frac{-6}{1^4} = -6$

$$p_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(1)}{k!}(x-1)^k$$

$$= \frac{f^{(0)}(1)}{0!}(x-1)^0 + \frac{f^{(1)}(1)}{1!}(x-1)^1 + \frac{f^{(2)}(1)}{2!}(x-1)^2 + \frac{f^{(3)}(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4$$

$$= \frac{0}{1}(x-1)^0 + \frac{1}{1}(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{6}{4!}(x-1)^4$$

$$= \boxed{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}}$$